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Intervention and Peace

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Intervention and Peace[☆]

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Abstract

Intervention often does not lead to peace, but rather to prolonged conflict. Indeed, we document that it is an important source of prolonged conflicts. We introduce a theoretical model of the balance of power to explain why this should be the case and to analyze how peace can be achieved: either a hot peace between hostile neighbors or the peace of the strong dominating the weak. Non-intervention generally leads to peace after defeat of the weak. Hot peace can be achieved with sufficiently strong outside intervention. The latter is thus optimal if the goal of policy is to prevent the strong from dominating the weak.

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1. Introduction

One of the facts of warfare is that victory in battle weakens the opposition making further victories easier. There are exceptions in conflicts over large geographical areas - for example the French and German invasions of Russia failed despite initial successes due to the overstretch of supply - but in regional conflicts over good terrain, absent outside intervention typically one side eventually achieves enough success that it is ultimately able to win the war. Outright victory tends to lead to peace, albeit the peace of the strong ruling the weak. Examples of this are the Union victory in the U.S. Civil War, the defeat of Napoleon and the defeat of Germany and Japan in the Second World War.

When there is outside intervention the weak may be propped up for a long period of time or even indefinitely leading to prolonged and often very bloody conflicts that may last decades or even generations. This depends in part on the goals of the outside powers and their strength. In many cases they support a balance of power, either for selfish reasons, to assure weak opposition, as in the case of Britain supporting a balance of power in continental Europe over many centuries, or because different outside powers take different sides in a conflict. Two obvious examples of conflicts prolonged over decades by outside intervention maintaining a balance of power are the Vietnam War, and the conflict between Israel and the neighboring Arab countries. By contrast conflicts without outside intervention - such as World War II or the US Civil war - are typically short, lasting on the order of five years before one side wins. We do not address the issue of the relative desirability of short term peace versus long-term conflict, but instead try to develop a useful model of the length and nature of conflict and how it depends upon outside intervention.

The model we develop is a stochastic model of regional conflicts. Under modest assumptions, absent outside intervention one side will win - and relatively quickly - leading to a hegemony and a long peace - that of the conqueror over the vanquished. By contrast, outside intervention typically supports the weaker side and can lead to a balance of power rather than a hegemony. By doing so it typically prolongs conflict. It does, however, protect the weak from the strong. Hence there is a trade-off: peace being desirable on the one-hand and the protection of the weak being desirable on the other. We find that when the latter is a priority the level of intervention is a relevant determinant of the nature and length of the conflict, with stronger intervention being generally preferable towards the goal of minimizing casualties.

We first develop a stochastic model of regional conflict, then we discuss the implications in a number of different conflicts historical and contemporary.

2. The Basic Model

We examine a conflict between two groups denoted by $j = 1, 2$ that we refer to as *societies* - these might be nation-states, non-governmental organizations, or bands of rebels.[3] These societies

[3] A related model with more societies can be found in Levine and Modica (2017).

compete in a particular region in which the absence of significant geographical or other barriers do not provide natural protection against enemies. For example, the model is not intended to apply to a mountainous region such as the Balkans where successful invasion and conquest is difficult. They compete over resources we call *land*, although it should be understood that land includes the people, factories and so forth on that land; this means we should think of units of land as measured by the resources they contain not by square miles, so for example a city block might represent a unit of land equivalent to a unit of land constituting many square miles of barren desert. Notice that we do not deal with situations where one group does not hold land or resources, but rather engages in violent activity such as terrorism then conceals itself within the broader society. At the moment we assume also that there are no outside influences: we will generalize the model to allow for this subsequently.

In total the region comprises $L \geq 3$ discrete units of land in total and the conflict takes place over time $t = 1, 2, \ldots$ We denote the land-holding of society j at time t by $L_{jt} \geq 0$. The time unit is chosen so that at most one unit of land can change hands in one period. When discussing society j we will typically refer to the opposing society by k.

Institutional Strength and Aggregate Power

The outcome of a conflict depends in part on the institutional strength of the societies. For example, in 1846 over a period of a few weeks thirty three people seized control of the state of California from Mexico. This was possible because Mexican institutions in California were exceptionally weak. We represent the institutional strength of society j by a coefficient $\gamma_j > 0$ we call *unit power* intended to capture the efficiency of a society in conflict. This includes the stability of the government, the ability to collect taxes and conscript soldiers - and depends in turn on the law-abidingness of citizens, the efficiency of the courts, and the overall economic strength of the society. There are different ways in which societies may generate unit power. For example, during the cold war the United States had low taxes capturing a small fraction of the resulting high GDP, while the Soviet Union had high taxes capturing a large fraction of the resulting low GDP: in different ways each generated substantial unit power.

The critical feature of the model - and reality - is that ability to prevail in conflict depends not only on unit power but also on total available resources that in our simple model are represented by land - meaning a larger population and typically a more powerful military. So holding more land makes it easier to prevail in conflict. Indeed, if China were to go to war with Hong Kong, despite the much higher per capita GDP in Hong Kong there is little doubt about the outcome of such a war - the vastly greater resources of China would bring the war to a conclusion in a matter of days if not hours. To capture this size component we introduce the notion of aggregate power: the *aggregate power* of a society j is given by $\varphi_{jt} = \varphi(\gamma_j, L_{jt})$ where $\varphi(\gamma_j, 0) = 0$ and the function $\varphi(\gamma_j, L_{jt})$ is increasing in both γ_j and L_{jt}: greater unit power and greater resources both increase aggregate power. To visualize one may think of the simple multiplicative linear form $\varphi(\gamma_j, L_{jt}) = \gamma_j L_{jt}$. As a society gains land it gains strength because it commands greater resources, and as it loses land it loses strength because it commands fewer resources.

Conflict Resolution and the Markov Process

Conflict is a risky business: weather, luck in warfare, heroism, being in the right place at the right time - all these things matter in determining the result of a conflict. To account for this we introduce a stochastic model: we denote by $1 \geq p(\varphi_{jt}, \varphi_{kt}) \geq 0$ the *conflict resolution function* - the probability that society j gains a unit of land from society k. Naturally $p(\varphi_{1t}, \varphi_{2t}) + p(\varphi_{2t}, \varphi_{1t}) \leq 1$ since only one society can gain a unit of land. In fact, we assume that the inequality is strict: there is a chance that nothing happens and that neither society gains a unit of land from the other.

The critical - and obvious - assumption is that $p(\varphi_{jt}, \varphi_{kt})$ is increasing in φ_{jt} and decreasing in φ_{kt}. The stronger is a society and the weaker its opponent the more likely it is to prevail and gain land. This is by no means a new idea: in military theory it is known as Lanchester's Law - although that refers to a deterministic differential equation rather than a stochastic process in which luck plays a role.

The conflict resolution function defines a Markov chain. Taking as the state variable the land holding L_{jt} of society j and letting $L_{kt} = L - L_{jt}$ be the land holding of the opponent we see that with probability $p(\varphi(\gamma_j, L_{jt}), \varphi(\gamma_k, L_{kt}))$ society j gains a unit of land to $L_{j,t+1} = L_{jt} + 1$ and with probability $p(\varphi(\gamma_k, L_{kt}), \varphi(\gamma_j, L_{jt}))$ it loses a unit of land to $L_{j,t+1} = L_{jt} - 1$ and with the remaining probability keeps the original amount of land $L_{j,t+1} = L_{jt}$. Indeed, this is a special type of Markov chain called a birth-death process.

The crucial feature of the model is that as L_{jt} grows the probability that it continues to grow goes up. That is, as society j gains land its aggregate strength increases, the aggregate strength of its opponent decreases and so the probability with which it wins land goes up. This says - in effect - that a balance of power between two societies is unlikely. Through random luck one society or the other will gain the upper hand, and when it does so it will not need so much luck to prevail over the other society resulting in a situation we refer to as *hegemony* - one of the two societies controls all the land. In a hegemony of j we have $L_{jt} = L$. We now introduce some formal tools for analyzing Markov processes which make this point more precisely.

Resistance

As one society becomes predominant - controls most of the land - the chances that the weaker society is able to wrest a unit of land in the face of overwhelming opposition becomes very small. For example, on December 2, 1913 in Alsace-Lorraine a shoemaker Karl Blank laughed at German soldiers, and was beaten and paralyzed. Subsequently there were protests of up to 3,000 people: needless to say the probability that this "rebellion" would succeed in wresting control of the Alsace-Lorraine region from the German Empire was very small and indeed it did not succeed. By contrast on October 8, 1911 in Wuhan China a rebel Sun Wu was seriously injured by a bomb he was building, resulting in his capture and imminent execution. To avert this, soldiers of the New Army in the local barracks mutinied. The probability that this "rebellion" would succeed in wresting control of China away from the Emperor was also very small - and indeed numerous similar revolts over the past decade had collapsed resulting in the execution or exile of their leaders. This one, however, succeeded and less than a year later the Empire had fallen.

The idea that prevailing over overwhelming opposition is very unlikely but never-the-less can happen is captured by a parameter $\epsilon > 0$ which is a "small" number: the probability of such events is positive but it should go to zero with ϵ. A simple way to formalize this is to assume that the probability of j losing land to k is given by $p(\varphi_{kt}, \varphi_{jt}) = p_0(\varphi_{kt}, \varphi_{jt})\epsilon^{r(\varphi_{jt}, \varphi_{kt})}$ where $r(\varphi_{jt}, \varphi_{kt}) \geq 0$ is called the *resistance* of society j to losing land to k and $0 < p_0(\varphi_{kt}, \varphi_{jt}) < 1$. Thus for $\epsilon > 0$ indeed every transition has positive probability; but if the resistance $r(\varphi_{jt}, \varphi_{kt}) > 0$ then $p(\varphi_{kt}, \varphi_{jt}) \to 0$ with ϵ - and the faster the higher r is. These events are interpreted as "very unlikely" since the idea is that we observe the process for small ϵ. On the other hand for transitions with zero resistance $p(\varphi_{kt}, \varphi_{jt}) = p_0(\varphi_{kt}, \varphi_{jt})$ so their probability remains positive in the limit - these are the "not so unlikely" events. So: if $r(\varphi_{jt}, \varphi_{kt}) > 0$ it is "very unlikely" that j loses land to k, while if $r(\varphi_{jt}, \varphi_{kt}) = 0$ that occurrence is in comparison "not so unlikely" for small ϵ.

With the notion of resistance we can apply powerful tools from the theory of Markov chains to analyze the conflict model when ϵ is small.

Hegemonic Resistance and Weak Geographic Barriers

Since the probability of winning land $p(\varphi_{jt}, \varphi_{kt})$ increases in own aggregate power and decreases in opponent aggregate power, we make the same assumption concerning the resistance to losing land: if resistance of j to losing land $r(\varphi_{jt}, \varphi_{kt})$ is positive it increases in own aggregate power and decreases in opponent aggregate power.

A hegemony once obtained will not persist forever. There can be internal dissension resulting in rebels seizing a unit of land. Since $\varphi(\gamma_k, 0) = 0$, if j has a hegemony the resistance to this is $r_j^h = r(\varphi(\gamma_j, L), 0)$, which we call the *hegemonic resistance.* We assume that the institutional strength of the two societies is great enough that they have positive resistance to losing land when they hold a hegemony: that is, we assume that $r_j^h > 0$ for $j = 1, 2$. Observe that this implies that in the limit as $\epsilon \to 0$ a hegemony, once entered, persists forever (it is absorbing); for small ϵ it will last for a long time, and we will investigate "how long" this will be.

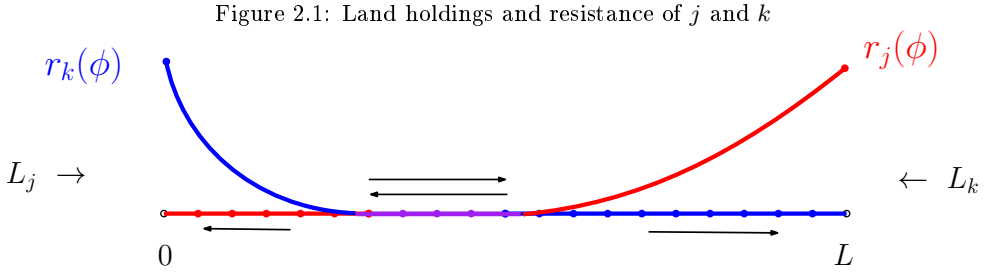
We also want to capture the idea that we are modeling a region within which geographical and other barriers are weak. We do so by assuming that the weaker society - unprotected by barriers as it is - has no resistance to losing land. When geographical barriers are strong it may be that neither of two nearly equal powers has a realistic chance of taking land from the other. It is no mystery why Switzerland is Switzerland or the Balkans are Balkanized - this is a matter of mountains and rugged terrain ill-suited for invasion. But our goal here is to study conflict that takes place in a region within which there are not important geographical barriers.[4] To capture this notice that monotonicity implies that if $\varphi_{jt} \leq \varphi_{kt}$ then $r(\varphi_{jt}, \varphi_{kt}) \leq r(\varphi_{kt}, \varphi_{jt})$, that is the weaker has lower resistance. We assume it has zero resistance:

[4]It is also the case that particular military technology may favor the defense so that even in the absence of geographical barriers invasion is impractical. However historical examples of this type are difficult to find. One example may be the stand-off between the Roman Empire and Persia: the two powers used incompatible military technologies, neither able to defeat the other.

Assumption 1. *If* $0 < \varphi_{jt} \leq \varphi_{kt}$ *then* $r(\varphi_{jt}, \varphi_{kt}) = 0$.

This implies that if the opposing society is at least as strong then the probability of losing land is positive as $\epsilon \to 0$. Notice that this also crucially implies that at any given configuration if j's resistance is strictly positive then k's resistance has to be zero. The typical resistance configuration is illustrated in figure 2.1.

Figure 2.1: Land holdings and resistance of j and k



Land of j is measured from the left, land of k is measured from the right. The dots correspond to the discrete units of land. Society j's resistance to losing land is in red, k's resistance is in blue. So j's red resistance decreases from right to left with j's land and once it becomes zero it remains there; and vice versa for k - the blue line decreases from left to right with k's land. The purple segment appears where both resistances are zero (red line superimposed on blue line). So for instance in the right part of the figure where j's resistance is positive and k's resistance is zero the system moves right - even more land for j - with positive probability for all $\epsilon \geq 0$, while the probability of moving left is zero in the limit; in the purple zone both societies have zero resistance to lose land and the system can go either direction with positive probability; this is indicated by the arrows. Notice that only hegemonies are absorbing in the limit process.

3. Stochastic Stability and Hegemony

We are now in a position to give a theoretical analysis of the Markov process. First, our assumptions guarantee that the Markov process is aperiodic and ergodic for $\epsilon > 0$, which in turn implies that there is a unique ergodic probability distribution μ_ϵ over the state space describing how frequently the different states are visited. From Young (1993) and the regularity conditions on resistances, we also know that as $\epsilon \to 0$ the ergodic distributions μ_ϵ have a unique limit μ_0 and that this limit places weight only on absorbing states for the process with $\epsilon = 0$. For simplicity we will refer to these states simply as *absorbing* with the understanding this is true only when $\epsilon = 0$. Those absorbing states that have positive probability in the limit distribution μ_0 are called *stochastically stable*: they represent the state or states which are observed "most of the time" when ϵ is small.

It should be apparent that when $\epsilon = 0$ the hegemonies of $j = 1, 2$ are absorbing: since the hegemonic resistance is by assumption positive when $\epsilon = 0$ the probability of leaving hegemony is also zero. By contrast, Assumption 1 implies that all the other states are transient when $\epsilon = 0$. The key idea is that the society that is weaker in aggregate power always has a positive probability

of losing land to the strong: this further weakens them and strengthens the opposition - so that there is a positive probability from any state L_j of reaching a hegemony of the society which begins with greater aggregate power.

Hence our first result, following from Corollary 5 Levine and Modica (2016) section 6.3, formally captures the instability of a balance of power between two societies:

Theorem 1. *The stochastically stable state of the system is a hegemony of the society with the highest unit power γ_j or if the two are equal, both hegemonies $j = 1, 2$ are stochastically stable.*

We can say a bit more than that: we can say relatively how likely the two hegemonies are. Let z_j denote the state in which j has a hegemony, that is $L_j = L$. If we compute the *relative ergodic resistance*

$$\rho(z_j, z_k) = \lim_{\epsilon \to 0} \frac{\log\left(\mu_\epsilon(z_j)/\mu_\epsilon(z_k)\right)}{\log \epsilon}$$

then the order of magnitude of $\mu_\epsilon(z_j)/\mu_\epsilon(z_k)$ is $\epsilon^{\rho(z_j, z_k)}$. To compute $\rho(z_j, z_k)$ we introduce the concept of *radius* which in the current context is the total resistance of the path from one hegemony to another. Starting the hegemony of j the resistance to losing one unit of land is r_j^h. Once the first unit of land is lost, the resistance to losing the second unit of land is $r(\varphi(\gamma_j, L-1), \varphi(\gamma_k, 1))$. We can continue, computing the resistance to losing each additional unit of land until we reach a hegemony of k. If we add together these number we can define the radius $R_j = r_j^h + r(\varphi(\gamma_j, L-1), \varphi(\gamma_k, 1)) + \ldots$ from a hegemony of j to a hegemony of k. From Levine and Modica (2016) we find that $\rho(z_j, z_k) = R_k - R_j$. That is, the relative frequency with which we see z_j to that with which we see z_k is to an order of magnitude ϵ raised to a power equal to the difference between the least resistance of going from z_k to z_j and that of going the opposite direction. The harder it is to go from one hegemony to the other, the more frequently we see that hegemony. In particular, the stable hegemony is the one with higher radius. [5] Moreover, the expected length of time before leaving the hegemony z_j for the other z_k is of order ϵ^{-R_j}.

Hegemonies are Common

Our theory says we should generally see hegemony. The idea of history being dominated by hegemonic states may seem a strange one, but with some important exceptions it is borne out by historical facts.[6] Take, for example, the largely geographically isolated region of China: bounded by jungles in the South, deserts on the West, cold arid wasteland in the North and the Pacific Ocean in the East. We find that during the 2,234 years beginning from when we have decent historical records in 221 BCE the area was ruled by a hegemonic state roughly 72% of the time, with five interregna. Less reliable records exist for the area of Egypt, but in the 1,617 years from 2686 BCE to the end of the new Kingdom in 1069 BCE we see hegemonic rule 87% of the time with two

[5] Except as otherwise noted the proofs of the theorems in this paper are applications of results in Levine and Modica (2016). A set of results for a related version of this model that allows for multiple societies can be found in Levine and Modica (2017).

[6] See Levine and Modica (2012) for data and sources.

interregna. In Persia during the 1,201 years from 550 BCE to 651 CE we see hegemony 84% of the time with two interregna. England has been largely hegemonic within the geographically confined area of the island of Britain for 947 years from 1066 CE to the present. The Roman Empire ruled the Mediterranean area as a hegemony for 422 years from the advent of Augustus in 27 BCE to the permanent division into Eastern and Western Empires in 395 CE and the Eastern Roman Empire lasted an additional 429 years until the advent of the Caliphate in 814 CE. The Caliphate itself lasted 444 years until the Mongol invasion in 1258. After a 259 year gap, the Ottoman Empire established a hegemony over the same general area for 304 years from the conquest of Egypt in 1517 CE to the Greek revolution in 1821 CE.

Hegemonies are not Ubiquitous

While hegemonies are common in history, there are two glaring exceptions: except for brief periods neither the subcontinent of India nor, following the fall of the Western Roman Empire, the area of continental Europe were subject to a hegemonic state: indeed the situation, especially in Europe, can better be described as a balance of power between competing societies. Clearly the theory is deficient: it says that as one side gains an advantage it becomes more likely to gain additional advantage. Evidentially this was not true in Europe and India. To see why this might be the case the example of the Korean war is useful. In September 1950 North Korea was on the verge of dominating the South. On the 15th of September the United Nations led by the United States launched an amphibious invasion reversing the situation. But rather than gaining resources and weakening the North Koreans the result of this success was the entry of China into the war on the side of North Korea - resulting in the United Nations forces being pushed back and ultimately a stalemate. The key point is: gaining land leads to greater weakness of the opponent only if it does not draw outside intervention.

The intervention of outsiders - protected typically by their own strong geographic barriers so not at risk in the conflict - is common in history. In Europe following the fall of Rome and up to around 1066 we have the continued interference of northerners - the Vikings and later Swedes were especially well protected by their own geography. Following 1066 we have the constant interference of England - also safe behind a water barrier: during this period we observe that England constantly intervened in continental conflicts but always to support the weaker side, and eventually this policy of balance of power became explicit.[7] India also was subject to repeated invasion from central Asia - protected not by water but by difficult desert and mountain terrain.[8] Of course China too was

[7] It is not completely correct to view England and Scandinavia as "outsiders" as at various time they had continental interests and conversely, but the key point is that they had a core area relatively safe from invasion. In a different direction Hoffman (2013) argues a role also for the Western Catholic church which in Europe acted as a balancing force much akin to to the outsiders of our model.

[8] The exact nature of the asymmetry in the physical geographical barrier is uncertain, but it is a fact that India has been invaded numerous times successfully from Central Asia, but there have been no successful conquests of Central Asia from India. Phil Hoffman in a private communication suggests that part of the answer may lie in the fact that the area of Central Asia is well suited for raising horses and India is not, and that horses play a central military role in conflict between Central Asia and India.

subject to outside influence - particularly that of the Mongols. However, the relative size of the Mongolia is quite small relative to China - less than half a percent of the population - while the population of Scandinavia was about 5% that of continental Europe, that in central Asia about 5% that of India, while England was about 8% of continental Europe.[9]

The role of England in maintaining a balance of power on the continent is well documented and notorious for its complete cynicism. From the rise of Spain following the discovery of America in 1492 through Brexit in 2016 British foreign policy has largely been aimed at preventing a hegemony over continental Europe. Many books (see for example Sheehan (1996)) have been written on the topic and few discussions of European history fail to remark upon the striking fact that Britain consistently changed sides in conflicts to support the losing side. Most dramatic perhaps is the shift to an alliance with France in 1904 in the face of the German threat. Note that until the advent of the European Union and the fall of the Iron Curtain this policy was quite successful. The latest effort to break up the continental hegemony of the EU - Brexit - may be less successful: contrary to the predictions of its advocates it seems to have strengthened pro-European sentiment on the continent.

To further advance the theory and understand the role of outside intervention in the balance of power we now extend the model to incorporate outside intervention

4. Outside Intervention and the Balance of Power

We introduce *outside forces* with *intervention power* φ_0. Outsiders are assumed to be protected by geography, climate or sheer strength from action by the region in question. One example is the aforementioned case of Great Britain with respect to continental Europe. Currently the U.S. and Russia are outside forces with respect to the Middle East, being protected by distance, the ocean (in the case of the U.S.) and by military strength from Middle Eastern societies.[10]

These outside forces can *reinforce* society j and we focus on the case in which only one party to the conflict is the beneficiary of outside intervention at any moment of time. We assume that these outside forces have a fixed power φ_0. If society j is reinforced its *combined power* is $\phi_j = \varphi_j + \varphi_0$, otherwise it is $\phi_j = \varphi_j$. Hence the conflict resolution is now given by $p(\phi_{jt}, \phi_{kt})$ rather than $p(\varphi_{jt}, \varphi_{kt})$, and the resistance function is also changed accordingly.

The behavior of outside forces is determined by an intervention policy which we initially take to be exogenous. We study a simple but important intervention: intervention on behalf of the weak.[11]

[9] Note that geographical factors matter in our argument only in so far as they give rise to outsiders who influence the evolution of the relationships between the other groups. An existing literature, including Diamond (1998), gives physical geography a direct role, arguing for example that the terrain of Western Europe is more defensible than that of China, hence less susceptible to hegemony. Besides this particular claim being challenged on physical grounds (Hoffman (2013)), such considerations have no bite in the Chinese case. Incidentally: while this discussion includes only the area of Europe, Asia and North Africa, it should be borne in mind that until modern times 90% of the world population lived in this area.

[10] Note that we are not considering here terrorism - which on the scale of conflict it is relatively minor.

[11] Intervention on behalf of the strong may hasten hegemony but will not lead to a balance of power.

Specifically we assume the there exist thresholds $\overline{L}_j, \overline{L}_k$ with $\overline{L}_j + \overline{L}_k < L$ and such that if $L_{jt} \leq \overline{L}_j$ then outsiders reinforce society j. The inequality $\overline{L}_j + \overline{L}_k < L$ means that $\overline{L}_j$ appears on the left of $\overline{L}_k$. Notice that we are agnostic about whether it is different or the same outsiders who reinforce j as who reinforce k; we discuss this further below when consider why outsiders might intervene. The assumption that the size of intervention φ_0 is the same on both sides is a simplification enabling us to focus on $\overline{L}_j$ as a measure of the strength of intervention.

It is useful at this point to denote combined power of j as $\phi_j(L_{jt})$ since unit power γ_j is determined by j while intervention is determined by L_{jt} (because $L_{kt} = L - L_{jt}$).

Stochastic Stability and the Balance of Power

In Appendix 1 we show that with outside intervention in addition to hegemonic states that are stochastically stable there may also be *balance of power segments* consisting of a contiguous collection of states. To analyze segments we extend the idea of an absorbing state to that of an *absorbing set* - meaning that as $\epsilon \to 0$ the probability of escaping from the set goes to zero, but the positive probability of moving about within the set remains positive. Hence segments are absorbing if at the left end j has positive resistance; at the right end k has positive resistance; and in the interior, if nonempty, both have zero resistance to losing land. From the general results of Young (1993) discussed above only absorbing segments can be stochastically stable.

There may be no absorbing states at all, a situation we refer to as *hegemony.* In addition two (and only two) types of absorbing segments are possible that we refer to as *hot peace* and *prolonged war.* In both cases either the left endpoint of the segment is the intervention threshold for j or the right endpoint of the segment is the intervention threshold for k.

First, let us explain why one of the endpoints must be an intervention threshold. Suppose the left endpoint is not an intervention threshold. One of the two societies must be weak and have zero resistance. If society j has zero resistance and there is no intervention threshold we move left without resistance: society j loses land so we are not really at the left endpoint of an absorbing segment. If, on the other hand, society k has zero resistance then as it loses land it continues to have zero resistance at least until the intervention threshold is reached. If at the intervention threshold the outsiders are strong enough to protect society k then this terminates the absorbing segment - we cannot escape to the right. If the outsiders are not strong enough then resistance is zero until hegemony is reached so there is no balance of power segment at all.

Second, the segment must either run the entire length between the intervention thresholds or it must have length one. To see this, suppose that the segment starts at the left intervention threshold. If moving a second step to the right has resistance then the segment has length one. If it does not have resistance then moving further to the right continues to have no resistance at least until we encounter the right intervention threshold and the outsiders come in to prop up society k. Put differently: either k is strong enough to defend itself at the left intervention threshold so the threshold is segment is short, or it is not, in which case the segment runs all the way until outside intervention occurs.

This gives rise then to a classification of absorbing segments:

Hot peace is a segment of length one at one of the intervention thresholds - that is either $L_j = \overline{L}_j, \overline{L}_j + 1$ or $L_k = \overline{L}_k, \overline{L}_k + 1$ (inclusive). Here a single unit of land changes hands back and forth. As well as a single hot peace segment there can be a pair of hot peace segments, one at each intervention threshold.

Prolonged war is a segment running from one intervention threshold to the other, that is from $L_j = \overline{L}_j$ to $L_j = L - \overline{L}_k$ inclusive. We also require that the intervention thresholds not be adjacent, $\overline{L}_j + \overline{L}_k < L - 1$, so that the segment is longer than one and is not a hot peace.

Hot peace, which to be clearly contrasted with the prolonged war case is modeled as a unit of land which changes hands back and forth. From a formal point of view if we were to subdivide the units of land the length of hot peace segments would shrink while the length of prolonged war segments would not. Hot peace can be thought of concretely as a relatively low key and "peaceful" conflict, with border skirmishes going on without land actually being gained or lost - for example, the recent conflict between Israel and Lebanon which occasionally flares into the firing of rockets over the border or a small border incursion. The case also covers situations with a demilitarized or neutral zone separating two opposing forces facing off against one another where again land does not actually change hands - as in the Cold War. As this paper is not about the tactics of military combat it is convenient to represent this as a single unit of land that changes hands back and forth although as a practical matter this may not be true. The point is that while a hot peace is not peace the level of conflict and casualties are low as the fighting is extremely limited.

The case of prolonged war is on the contrary a real, bloody war where the two sides fight back and forth losing and gaining substantial amounts of land and not merely skirmishing at the border. The civil war in Syria in the last years is a case in point.

Whether we see hot peace, prolonged war or hegemony depends on the strength of intervention. We distinguish four levels of intervention *on behalf of* j:

Definition 1 (Intervention strength)**.** 1. *Strong.* Intervention takes place when resistance is positive in the absence of intervention: $r(\varphi_j(\overline{L}_j), \varphi_k(L - \overline{L}_j)) > 0$

2. *Ineffective.* Intervention is insufficient to give positive resistance: $r(\varphi_j(\overline{L}_j) + \varphi_0, \varphi_k(L - \overline{L}_j)) = 0$; this includes the case where there is no intervention.

For the remaining cases we assume that #1and #2 do not hold, that is $r(\varphi_j(\overline{L}_j), \varphi_k(L - \overline{L}_k)) = 0$ and $r(\phi_j(\overline{L}_j), \phi_k(L - \overline{L}_j)) > 0$:

3. *Medium.* When j gains a unit of land above the threshold (thus losing support) the opponent has zero resistance to losing land: $r(\phi_k(L - \overline{L}_j - 1), \phi_j(\overline{L}_j + 1)) = 0$ and $\overline{L}_j + \overline{L}_k < L - 1$.

4. *Weak.* When j gains a unit of land above the threshold the opponent has positive resistance to losing land: $r(\phi_k(L - \overline{L}_j - 1), \varphi_j(\overline{L}_j + 1)) > 0$

Depending on the level of intervention some segments are absorbing and others are not. In Appendix 1 we characterize the relationship between intervention and the existence of absorbing segments of different types. The results are reported in the following

Theorem 2. *Existence, if any, of absorbing segments depending on the type of intervention on behalf of societies j and k can be summarized in the following table (where land is expressed in units of L_j):*

Table 1: Intervention and Peace

	strong k	medium k	weak k	ineffective k
strong j	impossible	impossible	hot peace at $L-\overline{L}_k$	hegemony of j
medium j	hegemony	prolonged war or hot peace at $\overline{L}_j = L-\overline{L}_k-1$	hot peace at $L-\overline{L}_k$	hegemony of j
weak j	hot peace at $\overline{L}_j$	hot peace at $\overline{L}_j$	hot peace at both $\overline{L}_j$ and $L-\overline{L}_k$	hot peace at $\overline{L}_j$
ineffective j	hegemony of k	hegemony of k	hot peace at $L-\overline{L}_k$	hegemony

We are also interested in whether these absorbing segments are stochastically stable. As might be expected this depends on the strength of the outside forces. In Appendix 2 we prove the following:

Theorem 3. *There exist $\infty \geq \overline{\varphi}_0 \geq \underline{\varphi}_0 > 0$ such that if $\varphi_0 > \overline{\varphi}_0$ and if intervention thresholds are positive on both sides there are stochastically stable balance of power segments but not stochastically stable hegemonies, while if $\varphi_0 < \underline{\varphi}_0$ there are stochastically stable hegemonies but not stochastically stable - or even absorbing - balance of power segments.*

Analysis of the relative persistence of the absorbing states in the presence of balance of power segments is given in Appendix 3.

5. War and Peace

Theorem 2 shows that there is a non-monotonicity in the consequences of intervention. Ineffective intervention, not surprisingly, leads to peace in the form of hegemony. Weak intervention leads to hot peace in which a weak opponent is propped up but can make no headway against a strong opponent. An example of this is the intervention of the United States on behalf of the Northern League in Afghanistan prior to 9/11. As the strength of intervention increases we have the possibility of more serious conflict - prolonged war - but eventually this goes away and conflict subsides.

To understand this non-monotonicity it is useful to consider a simple case. Suppose that j and k are equally strong so that $\gamma_j = \gamma_k = \gamma$, and that the intervention policy is symmetric so that $\overline{L}_j = \overline{L}_k$. Hence intervention policy is indexed by a single scalar, the land threshold for intervention on behalf of both contenders. We assume that the number of units of land L is odd.[12] Finally we assume that the strength of the intervenor(s) φ_0 is high enough that strong intervention is possible, but that it is ineffective for $\overline{L}_j$ sufficiently small.

Here we increase the intervention threshold for both sides at the same time. Start with $\overline{L}_j$ small. In this case as we have noted intervention is ineffective - there is no point in intervening

[12] So it is feasible for the two thresholds $\overline{L}_j, \overline{L}_k$ to be adjacent; this would be ruled out by symmetry if L is even.

when j has become so weak that they have lost even with outside help. In this case there is no balance of power segment, but rather a hegemony of one society: we refer to this as the *peace of the strong over the weak*. As $\overline{L}_j$ increases, eventually the point is reached where intervention is weak. As we indicated we now have a hot peace in which the weaker side survives by virtue of outside support and the stronger side by virtue of their strength.

The key transition to understand is that from weak to medium intervention - because it is medium intervention that leads to a prolonged war. Why is this? As the intervention threshold increases the side receiving support is propped up when it is relatively strong: eventually strong enough that the opposition no longer has resistance to losing land. At this point intervention becomes medium and when launched from behind the shield of foreign protection success is now possible and may sometimes range until intervention occurs on the side of the opponent. As an example of this we might consider the second Vietnam war until the withdrawal of the United States in 1973: here we have the United States intervening to prevent the fall of South Vietnam and the Soviet Union intervening to prevent the fall of North Vietnam. The war ranged for nearly twenty years with substantial battle deaths and loss and gain of territory on both sides and no doubt would have gone longer had the United States not withdrawn its intervention.

As the strength of intervention $\overline{L}_j$ increases further the length of the prolonged war segment shrinks reducing the scale of the conflict until eventually $\overline{L}_j$ reaches the center and we are again at hot peace. As an example of this we might consider the intervention of the United States on both sides of the Israel/Egypt conflict at the Camp David accords in 1978: in effect the United States provides arms and support to both armies to stare at each other across a border that will bring quick intervention in response to a violation.

We want to emphasize the non-monotonicity of the consequences of intervention in its strength: a weak or strong intervention leads to hot peace, but a medium intervention leads to prolonged war and it is the costliest in terms of lives and distress to the peoples and economies involved.

We may ask: why do we see prolonged war at all? Should not the participants knowing that fighting will simply rage back and forth between the intervention thresholds just skip the conflict? In an evolutionary model such as this there is in fact a prisoner's dilemma involved. If one side decides not to fight then it has no resistance to losing land but can never win it back, so it quickly winds up permanently at the intervention threshold. A society that will not fight will be whittled away by a society that will fight by losing land, but never gaining it. In fact we do not see societies like this - probably because without intervention they will cease to exist. We note that even very liberal modern democracies are quite aggressive in this respect: witness that Spain to this day lays claim to Gibraltar - a bit of land that has been British for over three centuries.

In addition to conflicts corresponding to absorbing states when $\epsilon = 0$ we can have *transitional wars* corresponding to movement between different absorbing states - that is, from hegemony to balance of power and back, or between different balance of power segments. From Levine and Modica (2016) we know that these conflicts must be relatively short in the sense that the expected length of time before an absorbing configuration is reached is bounded independently of ϵ while the

length of time spent in an absorbing configuration grows without bound as $\epsilon \to 0$.

6. Balance of Power and Modern Conflict

We shortly turn to details of different configurations and illustrate them with examples. In addition to discussing specific cases, we gather the substantial post World War II conflicts in the form of tables. Cases where one combatant did not occupy any land are excluded as the theory does not apply. For the rest we examined each postwar conflict in the Uppsala database. We excluded those marked as insignificant, those involving military coups, those involving invasions of minor powers by major powers (for example: 1956 invasion of Hungary) and guerrilla conflicts where the guerrillas did not control land and resources (for example: the Basque region). We examined each remaining conflict and believe that we have included the most significant. In some cases there were several intervention regimes: we discuss those separately. The data about individual conflicts is taken from Wikipedia. The tables show the region, the year in which the conflict began, and the number of years it lasted. Casualties (including civilian casualties) are reported in deaths per 100,000 per year which is the standard unit for reporting, for example, murder rates.[13] To put these numbers in context, note that the overall murder rate for Europe and Asia is about 3, for the entire world about 6, and for Africa about 12 and for the Americas about 16. So, for example, the death rate of 20 in the Sri Lankan civil war (a hot peace) is comparable to the murder rate in the Americas, while the death rate of 380 in the Syrian civil war (a prolonged war) is more than an order of magnitude higher. Following the casualty rates we list the parties and outside intervenors. In cases when war ended due to the withdrawal of intervention we report the "collapse" as the number of subsequent years until one side achieved victory. Entries in the table are arranged in chronological order.

Before examining the details, it is worth taking a broad overview of our findings. Intervention that either is designed to preserve the balance of power or which does so because of conflicting interests of the intervenors can lead either to a hot peace or a prolonged war. There is a large discontinuity in the amount of harm done in a hot peace and a prolonged war: in a hot peace death rates are on the order of relatively high murder rates, or in some cases lower, while in a transitional or prolonged war they are an order of magnitude larger then very high murder rates. Taking the Sri Lankan civil war as an example of hot peace we see that for 26 years the death rate was about 20, comparable to the highest murder rates in the world. Taking the breakup of India and Pakistan after the British withdrawal as an example of a transitional war it was vastly bloodier - the death rate was about 250. However, the Sri Lankan civil war lasted 26 years so the total is about 520, more than double that in India and Pakistan where the transitional war lasted only a year. Overall a hot peace does not seem to represent much of a savings in terms of casualties over non-intervention and a transitional war - but it does protect the weak. From a

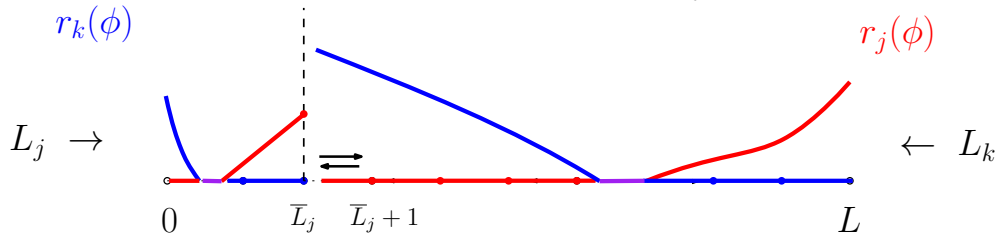
[13] Civilian casualties are the bulk of casualties and there are a wide range of estimates. We used the middle of the range of estimates.

policy point if we were to take the point of view that, say, Lebanon posed a threat, then keeping it a bloody mess for three decades would surely neutralize that threat - but from a humanitarian point of view it represents a catastrophe. If we are to take a very cynical view of the conflict between Shia and Sunni, especially the current hot war in Syria, as a Western effort to preserve a balance of power that neutralizes the Arab world as a threat - the wave of refugees descending on Europe with the consequent social and political problems shows that such an effort can have unintended consequences.

6.1. Weak Intervention and Hot Peace

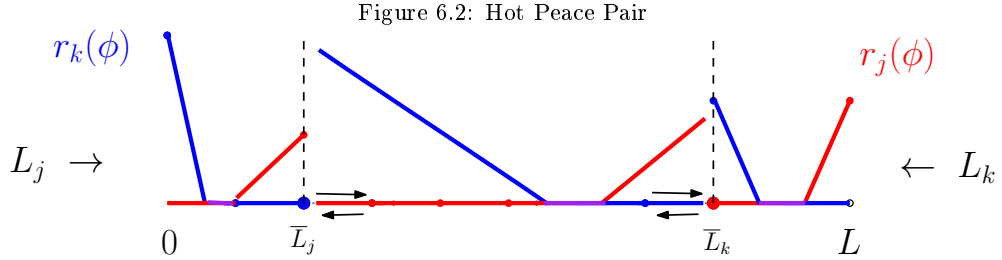
Theory. Weak intervention leads to hot peace: a single unit of land that changes hands back and forth always at the threshold for intervention. The intervenor prevents "their side" from losing, but is unwilling to help them win. This is illustrated in figure 6.1. These hot peace segments can also occur in pairs as illustrated in figure 6.2. In the discussion that follows we confine attention to the case of a single segment as it is the relevant one in practice.

Figure 6.1: Single Hot Peace from $\overline{L}_j$



Land of j is measured from the left, land of k is measured from the right. The dots correspond to the discrete units of land. Society j's resistance to losing land is in red, k's resistance is in blue. The purple appears where both resistances are zero (red line superimposed on blue line). Resistance decreases with land holdings, and when it is zero the system moves with positive probability in the direction of no resistance. The back and forth arrows indicate the location of the balance of power segment. Notice how the resistance of j (red) jumps up and that of k (blue) jumps down (to zero since only one resistance can be positive) at the intervention threshold where outsiders intervene on behalf of j (red). The arrows indicate that from $\overline{L}_j$ to $\overline{L}_j + 1$ and back there is zero resistance.

Figure 6.2: Hot Peace Pair



Land of j is measured from the left, land of k is measured from the right. The dots correspond to the discrete units of land. Society j's resistance to losing land is in red, k's resistance is in blue. The purple appears where both resistances are zero (red line superimposed on blue line). Resistance decreases with land holdings, and when it is zero the system moves with positive probability in the direction of no resistance. The back and forth arrows indicate the location of the balance of power segments where the system moves with no resistance. Discontinuities in resistances again appear at the intervention thresholds.

Discussion. Hot peace with weak intervention is matched against the strength of the opponent: one example is in Afghanistan, which has had varied intervention policies over the years since civil war begin in 1978. Initially the Soviet Union intervened and a prolonged war resulted until the Soviet withdrawal in 1988. What followed seems to have been a hot peace where intervention was matched against the strength of the opponent: the Taliban overran most of the country, but a small enclave remained under the control of the Northern League propped up by foreign support. This persisted until the strong United States intervention in 2001 resulting in the prolonged war that continues to this day. More common examples of weak intervention are places where the British prop up small enclaves such as Gibraltar or the Falklands (Malvinas). As can be seen in Table 2, while hot peace can last a long time casualties are minimal.

6.2. Medium Intervention and Prolonged War

Theory. Medium intervention on both sides results in prolonged war. This is illustrated in figure 6.3 below. Notice that according to the definition there is positive resistance to leave at the two extremes and zero resistance to move in the interior.

Discussion. As we have indicated, this seems the least justifiable form of intervention. Data are collected in Table 3. The only rationale for medium intervention we can think of is that a region poses a particular danger and hence the importance of keeping it weak offsets the bloody harm of prolonged war. Yet, if we look at the record, Vietnam, Sudan, Angola, Lebanon, and Syria do not appear to have ever presented any great danger to the intervening powers. It is interesting that while the US intervention in Vietnam is widely criticized outside the US, it seems to be so for mostly the wrong reasons. Surely there was nothing wrong with supporting the South, for, despite all the shortcomings of its government, there was no popular desire to be ruled by the equally bad or worse government in the North. Nor can there be much moral doubt about opposing the spread of communism: one need not look further than North Korea and Cuba - two of the most miserable places in the world - to see that. Nor is it clear why the direct involvement of the US is worse

Table 2: Weak Intervention

Region	Start	Duration	Casualties	Parties	Intervenors	Collapse
Gibraltar	1713	304	0	Spain Gibraltar	Britain	
Falklands	1833	184	0	Argentina Falklands	Britain	
Hong Kong	1842	155	0	China Hong Kong	Britain	0
Sri Lanka	1983	26	20	Government Tigers	India[1]	
Afghanistan - Northern Alliance	1994	7	5 [2]	Taliban Northern Alliance	United States	0.2

Table Notes

1. Indian troops left Sri Lanka in 1990 nineteen years before the Government victory. Prior to sending troops in 1987 the Indian government aided the Tigers through the intelligence agency RAW. It likely that this continued after withdrawal in 1990, but there is no information available about this, so we cannot say when or even if India stopped supporting the Tigers.
2. Casualties in the war seem mostly to have involved Taliban massacres of civilians. The number are unclear but do not seem to exceed 10,000 which is the basis of our estimate of 5 per hundred thousand per year.

than indirect Russian involvement. From our point of view the US should be rather criticized for creating a prolonged and costly conflict by attempting to maintain a balance of power in the South.

6.3. Strong Intervention and Hot Peace

Theory. With strong intervention we get a hot peace in which a single unit of land changes hands between adjoining thresholds for intervention. This is illustrated in figure 6.4.

Discussion. There are two types of hot peace: a one-sided hot peace where weak intervention is matched against the strength of the opponent and a two sided hot peace where two outside intervenors stare eye-to-eye across a border. Significant cases are listed in Table 4. A classical example of strong intervention and the eye-to-eye stare is the Iron Curtain, where military forces of the intervenors - the US on the West and Soviets on the East - sat for decades eye-to-eye in the literal sense. No land changed hands, but this possibility is explicitly covered by the model. A more interesting case is in the Sinai where the intervention on both sides is by the United States. Naturally we do not see US soldiers staring at each other eye to eye across the border, but the essential element of the Camp David accords was the promise of substantial military support (in the form of equipment and training) for both sides. We do note that in some places outside intervenors in the form of UN blue-helmets patrol the boundary - taking both sides in effect - but their job is merely to act as monitors - they have not even enough military strength to protect themselves should a shooting war break out.

6.4. Other Types of Episodes

The model does not and is not intended to cover all possible types of conflicts.

Table 3: Medium Interventions with Prolonged Wars

Region	Start	Duration	Casualties	Parties	Intervenors	Collapse
Vietnam	1955	20	170	North South	Soviets US	1
Sudan	1955	60	330	North Sudan Southern Sudan	Egypt Ethiopia/Uganda	
Angola	1975	27	86	MPLA UNITA	Soviets South Africa	1 [1]
Lebanon	1975	31	400	Shia Christian/Druze	Syria Israel	1
El Salvador	1979	12	138	Government FMLN	United States Soviet Union	
Syria	2011	5+	380	Government Insurgents	Russia, Iran West	

Table Notes

1. The date at which intervention on behalf of UNITA ceased is unclear. We dated it to May 2001 when DeBeers - the main source of funding and illicit weapons shipments to UNITA - ceased operation in Angola.

Table 4: Significant Modern Hot Peace Episodes with Medium Intervention

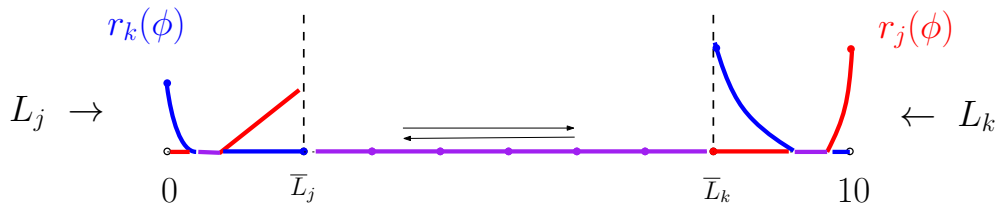
Region	Start	Duration	Casualties	Parties	Intervenors	Collapse
Iron Curtain	1945	46	0	Eastern Europe Western Europe	Soviets US	1
Sinai	1948	68+	1	Israel Egypt	West Soviets [1]	
Korea	1950	66	8	North South	China US	
Nagorno-Karabakh	1988	28	26 [2]	Armenia Azerbaijan	Turkey Russia	

Table Notes

1. It should be noted that originally the Soviets supported Israel.

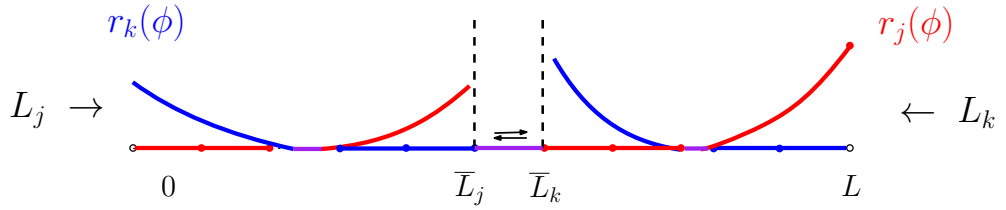
2. It is unclear in which population the casualties occurred. Virtually all deaths occurred during the six years of active war beginning in 1988. It is estimated that 28,000–38,000 died in that conflict. The population of Nagorno-Karabakh is only 147,000, but it is highly unlikely the bulk of casualties occurred among that population. We used the average of the population of Azerbaijan and Armenia as our base population.

Figure 6.3: Prolonged War



Land of j is measured from the left, land of k is measured from the right. The dots correspond to the discrete units of land. Society j's resistance to losing land is in red, k's resistance is in blue. The purple appears where both resistances are zero (red line superimposed on blue line). Resistance decreases with land holdings, and when it is zero the system moves with positive probability in the direction of no resistance. The back and forth arrows indicate the location of the balance of power segment where the system moves with no resistance. Discontinuities in resistances appear at the intervention thresholds.

Figure 6.4: Strong Intervention



Land of j is measured from the left, land of k is measured from the right. The dots correspond to the discrete units of land. Society j's resistance to losing land is in red, k's resistance is in blue. The purple appears where both resistances are zero (red line superimposed on blue line). Resistance decreases with land holdings, and when it is zero the system moves with positive probability in the direction of no resistance. The back and forth arrows indicate the location of the balance of power segment where the system moves with no resistance. Discontinuities in resistances appear at the intervention thresholds.

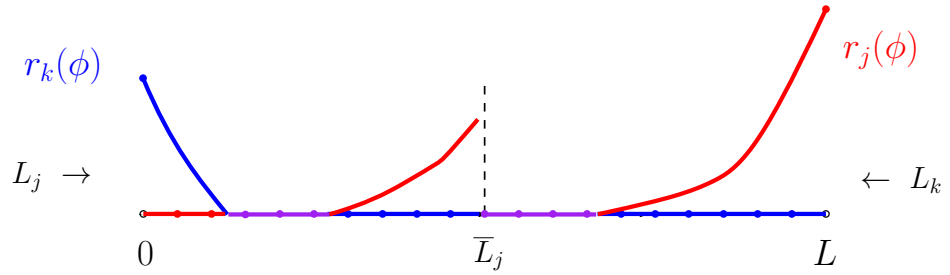
Anarchy. Our working assumption has been that hegemonic resistance is positive, but institutions may be so weak that hegemony is not absorbing. In the absence of outside intervention there are also no absorbing balance of power segments. In this case the theory predicts continual strife - at least until some group develops institutions adequate to persist. The situation in the Central African Republic, Somalia and Yemen appear to fall into this category. In the case of Somalia we quote from the US State Department website "There is no organized system of criminal justice, nor is there any recognized or established authority to administer a uniform application of due process. Enforcement of criminal laws is, therefore, haphazard to nonexistent."

Transitional Wars. Transitional wars are the fast leap from one absorbing set to another: from hegemony to hegemony, from balance of power to hegemony, from hegemony to balance of power or from one balance of power to another. The most important case is when there is no outside intervention or outside intervention is too weak to matter. In this case we have only the transitional

war of moving from one hegemony to another, or an unstable state where two societies hold land followed - relatively quickly - by the victory of one resulting in hegemony. However, transitional conflicts also occur with outside intervention in which the intervenor is attempting to help one side win and not to preserve a balance of power.

The case in which there is no absorbing segments and movement is directly from one hegemony to another is illustrated in figure 6.5.

Figure 6.5: No absorbing segment



Land of j is measured from the left, land of k is measured from the right. The dots correspond to the discrete units of land. Society j's resistance to losing land is in red, k's resistance is in blue. The purple appears where both resistances are zero (red line superimposed on blue line). Resistance decreases with land holdings, and when it is zero the system moves with positive probability in the direction of no resistance. Discontinuities in resistances appear at the intervention thresholds. For illustrative purposes we have taken the intervention threshold for k to be zero.

Indeed: we observe a number of wars with little outside intervention: these generally result in hegemony and moreover, as the theory predicts, they are relatively short. See Table 5. The only one longer than 1 year is the Iran/Iraq war. Often these transitional wars occur after a change in intervention policy in the form of the withdrawal of outside forces leading to collapse of the balance of power. A number of well known conflicts have this character: after the British withdrawal from Palestine in 1948 war broke out between Israel and the Arab nations: this lasted less than a year. Similarly when Britain withdrew from India conflict broke out between India and Pakistan including conventional warfare over Kashmir. This lasted slightly longer than a year. About a year after Richard Nixon agreed to "peace with honor" in 1973 - meaning actually that he agreed to stop intervening - North Vietnam launched an assault on the South winning the war in about a year. In Eastern Europe after Gorbachev announced that the Soviet Union would cease intervention the fall of the Berlin wall came about in a matter of months.

The Rwandan Civil War is an interesting case study in what happens without intervention. The conflict was largely ethnic between Hutu and Tutsi. On April 6, 1994 the plane of the Hutu President Habyarimana was shot down initiating the Hutu genocide perpetrated by the Tutsi. By July 3, 1994 - that is only about three months later - the Tutsi RPF overran the capital city of Kigale effectively ending the war. The absence of western intervention in this case is well known and usually discussed in the context of preventing the genocide. Given the time-line this was probably not feasible: the rapid forms of western intervention - air power, special forces - are

Table 5: Significant Modern Transitional Wars

Region	Start	Duration	Casualties	Parties	Intervenors	Collapse
India	1947	1	250	India Pakistan		
Palestine	1948	0.75	700	Israel Arab		
Bangladesh	1971	1	100	Bangladesh Pakistan	India[1]	1
Iran/Iraq	1980	8	100	Iran Iraq		
Falklands	1982	0.2	0	Argentina UK		0.2
Rwanda	1994	0.25	6800	Hutu Tutsu		0.25

Table Notes

1. India did not attempt to preserve a balance of power but helped Bangladesh to win the war.

ineffectual against large groups of people welding machetes, and by the time massive numbers of ground troops could have been put in place it would have been far too late. Rather the lack of western intervention is a case study in how prolonged war can be avoided: the Tutsi won and the peace of the strong over the weak has prevailed since.

We want to emphasize just how short are transitional wars compared to the prolonged conflict brought about by insufficiently strong outside intervention. The transitional war that brought peace to Rwanda lasted months. Moreover, the length of wars appears largely unrelated to whether they are civil wars: although most prolonged wars are civil, absent outside intervention they tend to be relatively short, if not so short as in Rwanda. Reaching farther back in history, the U.S. Civil War, bloody as it was, lasted only four years. World wars - in which outside intervention is not possible more or less by definition - also have been relatively short: four years in the case of World War I and six years in the case of World War II.

The overall point is that these transitional wars are short: less than a decade in length and often lasting only months. Hence although they are bloody, because they are short they are not necessarily more bloody than a hot peace that lasts many decades.

Balkanization. Our theory deals with only two contestants: but there are parts of the world that are Balkanized in the sense of the Balkans - or perhaps today the area of Syria and Iraq - where more than two contestants view in a sort of free-for-all. While the details are beyond this paper and can be found in Levine and Modica (2017) we can give the general idea.

In some areas such as the Balkans there is difficult terrain so that it is hard for any side to prevail. If every society has positive resistance to losing land all states will be absorbing (at $\epsilon = 0$ as usual). Since positive resistance events do occur over long periods of time, this means that there will be recurrent conflicts. This seems a relatively good description of the Balkans, which has been Balkanized and in a more or less perpetual state of conflict since about 1200 BCE with the

exception of periods of time when strong outside powers (Rome, the Ottomans) imposed peace. Each time, with the withdrawal of the outsiders low level periodic conflict seems to have more or less immediately resumed. Most recently large parts of the Balkans have been absorbed into the EU which may play much the same role as the Romans and Ottomans in bringing peace to the region - what will happen at some future time when that power should be withdrawn we suspect will be a resumption of the old divisions.

What about the case where terrain is good? Suppose we have a balkanization of more than two societies. We continue to maintain the assumption that the weakest society has no resistance to losing land. If outsiders cannot intervene simultaneously for both of the two weakest societies then one of them must have zero resistance to losing land, and we would expect there to be zero resistance to the strongest society winning the land. Such a change does not affect which are the two weakest societies: so collectively the two weakest societies always have zero resistance to losing a unit of land until one of them vanishes. Eventually we get down to two societies and the model we have developed above.

If there is a single outside intervenor this makes perfectly good sense - it is not very practical in a multi-way conflict to consistently intervene on behalf of two different clients. However, with more than one outside intervenor it may be that each intervenes consistently on behalf of a different client. Specifically: if outside intervention consistently takes place on behalf of both of the two weakest societies and is sufficiently strong as to give both positive resistance to losing land then it is easy to construct intervention policies that preserve the balkanization: as soon as one of the two weakest societies wins a unit of land intervention is withdrawn until it loses the land again. The key point is that we can have stability in a balkanization provided that there is more than on outside intervenor against the weak parties, but it is not so likely with just one.

The Middle East is the most obvious example of a region which is generally perceived as a balkanization. Never-the-less it can be usefully analyzed by breaking it down into two sub-regional conflicts: Israel versus Egypt in the South and Sunni versus Shiite in the North. In the South Israel and Egypt have intermittently fought from 1948 (the Arab-Israeli war listed in our table) to 1973 (the Yom Kippur war) after which peace negotiations began and ended with a further hot peace in 1979. This hot peace, enforced by strong US intervention on both sides takes the form of a treaty that has largely resulted in the cessation of bloodshed. This appears to be the most desirable form of hot peace.

The situation in the North is on the other hand a story of insufficiently strong intervention and bloody, prolonged wars. We have first the Lebanese civil war, then the Iran-Iraq war, then the conquest of Kuwait, followed by the liberation, the second Iraq war, and now by the Syrian civil war. As can be seen in our table some of these conflicts can be broken into separate regions which can be usefully analyzed by our methods.

The Kurds form a particularly interesting sub-case in the North. We have assumed that the weakest power is the most likely to lose land. But in a multi-lateral conflict the larger powers may be so focused on fighting each other that a smaller power is able to survive "in the shadows" so to

speak. Originally the Kurds were able to occupy land as a consequence of the civil disorder and no fly zone that followed the Iraqi defeat in Kuwait: Saddam Hussein's Sunni forces were tied up with defeating the Shia near Basra and especially the marsh Arabs. Following second Iraq war politically the Sunni's and Shia's were more concerned with each other than with the Kurds - who also received limited US support, very limited on account of the alliance with Turkey: Indeed the official US position has always been that Kurdistan should be part of a unified Iraq.

Syria has high resistance to losing land, especially with Russian support. The other major party to the conflict in Iraq and Syria, the ISIS, appear to be zealots, unstable, having no resistance to losing land and who consequently must either win quickly or vanish forever. As they have not won quickly we expect they will shortly vanish forever except perhaps as a rump group of stateless terrorists like their predecessor Al Quaeda. ISIS land, our model predicts, can go to either Syria/Iraq or to the Kurds. Then there are two possible scenarios:

(1) The US withdraws support for Kurds. In this case at some point Kurds will lose land to Syria and Iraq and Kurdistan will vanish;

(2) The US holds its support for Kurds. In this case our model predicts a balance of power between Kurds and Syria - but reality is a little different: owing to US concern with Turkey, sustained US support for Kurds is possible only if the Kurds compromise on their requests for an independent state and commit in advance to be part of a united Syria (like the Iraqi Kurds in Iraq) when ISIS is defeated. If this is the case - and we think it indeed is the more likely scenario - the outcome may be a balance of power within the context of particular states.

7. Why Intervene?

So far we have viewed the intervention thresholds as exogenous: in fact they are endogenously determined by the motivation of the outsiders. There are two elements: the cost of intervention and the motivations of the intervenors. Morevoer, the outside forces might be a single country or two external powers with possibly opposed interests. Both cases have concrete examples. As we have already mentioned, England in continental Europe has played the role of a single intervenor in favor of the weaker sides quite independently of their nationality. In the case of the United States and Russia in the Middle East the situation is obviously different, with each external power supporting a different side.

Benefits of Intervention

We suggest four possible benefits of intervention. One is to prevent hegemony that may be threatening. Examples of this are Britain and the balance of power on the continent and quite likely the US and USSR keeping Europe and especially Germany divided after WWII. It may also be that conflict among enemies is the preferred option since enemies who are fighting with each other are less threatening than enemies that are at peace with each other. Again: in the case of Britain and the balance of power this seems to have been the case.

Another is to extract economic rents - this may have been the case with the British Indian Empire - although the economic history literature seems divided on whether they managed to do so or not. Another purely economic motivation may be the desire to sell arms to the conflicting nations. Somewhat in between keeping an enemy weak and economic exploitation is the desire to extend military influence and provide a buffer against more distant rivals - for example Russia seems to have intervened in Syria in order to preserve and expand their naval base in Syria. Similar motivation may be territory for the pure sake of territory - the "manifest destiny" of the United States seems to be of this sort. Domestic popularity may also play a role. The Falkland Island war was driven by the lack of popularity of the Argentine junta while European intervention in Libya and Syria was also influenced by the unpopularity of Cameron and Hollande.

It is sometimes argued that the purpose of intervention is to preserve trading partners. However, this seems to make little sense: why not allow one side a quick victory then trade with the winner?

Intervention may also be driven by more moral considerations - although evidence for this is weak - either to keep your side from losing ("protecting democracy") or simply because of a desire to for the social good for peace - the motivation for this volume.

Costs of Intervention

The cost structure of intervention is relatively simple, but it is important that cost is not monotone. No intervention has no cost, but weak intervention - intervening when your side is already weak and the other strong - is more costly than medium intervention - when your side is relatively strong.

For any particular goal the most cost effective form of intervention should be chosen. Clearly there is no point in wasting money on intervention so weak to be ineffective so the practical choice is between weak, medium and strong intervention.

A simple and cost effective form of intervention is simply to provide your side with nuclear weapons - these are cheap and certainly a strong deterrent. Indeed this is exactly what we see. *De facto* the United States has provided nuclear weapons to Western Europe, Turkey, Japan and South Korea. By which we mean it stores arsenals of nuclear weapons in those countries. But *de facto* that puts control of those weapons in the hands of those countries. There is no sensible way in which a base containing nuclear weapons can be defended from the country hosting that base. As the infantry general Faceman replied to the air force general Turgidson in Dr. Strangelove: "General Turgidson, with all due respect for your [airbase] defense team, my boys can brush 'em aside without too much trouble." This was put to the test during the 1985 confrontation between US base defense forces and the Italian army at the Sigonella base over the custody of terrorists that had been captured by the US forces. Needless to say the Italian army won that. We should note also that it is unlikely that security precautions, activation codes and so forth are effective. It is one thing to protect a nuclear weapon from a terrorist with a screwdriver - it is quite another to protect it from the top scientists and laboratories in a sovereign state.[14] Finally: from the point

[14]Even if the original bomb cannot be used nuclear bombs are easy to build - the hard part is getting bomb grade

of view of deterrence it does not have to be a certainty that the country will be able to use US stored nuclear weapons - a strong possibility is likely to give an enemy pause. To quote again Dr. Strangelove: "Deterrence is the art of producing in the mind of the enemy... the FEAR." It is beliefs that matter.

Deterrence in general is widely used as a cheap form of intervention through a tripwire effect. The classical tripwire is the conversation between the British general Wilson and the French general Foch described in Barbara Tuchman's *The Guns of August:*

> A question that Wilson asked of Foch during his second visit in January 1910 evoked an answer which expressed in one sentence the problem of the alliance with England, as the French saw it.
>
> "What is the smallest British military force that would be of any practical assistance to you?" Wilson asked.
>
> Like a rapier flash came Foch's reply, "A single British soldier—and we will see to it that he is killed."

It has been argued that US participation in NATO is of a similar quality: that the actual NATO forces deployed during the cold war stood little chance against the Warsaw Pact so that their real purpose was for their defeat to trigger the use of nuclear weapons. To quote the conclusion of a the CBO report of July 1978 *Assessing the NATO/Warsaw Pact Military Balance* "The brighter assessments [of NATOs chances] are optimistic only in comparison with the more pessimistic ones."

Of course the downside of the tripwire effect is that the wire may be tripped and the cost may then be astronomically high as it was in World War I.

Single Intervenor

The most cynical type of intervention is that of a single intervenor attempting for strategic reasons to prevent hegemony or worse trying to stir up conflict to keep rivals weak. This may also be the case when two ostensible rivals collude: We may well ask about the motivation of the US and USSR in the cold war: were the two implicitly colluding to keep Europe weak and divided? In the case of Germany there is no doubt of the answer to that question.

We now introduce some explicit considerations. Let us rule out strong intervention as intervention on both sides is impossible, so we assume the intervenor chooses between no, weak and medium intervention with no intervention having cost zero, weak intervention cost 1 and medium intervention cost $0 < c < 1$. On the benefit sides we assume that the payoff to hegemony of either side is 0 and denote by w the benefit of prolonged war. We assume moreover that the benefit of a hot peace is $x < w$. Hence - with one player choosing both the row and column - that is whether to intervene on either or both sides - the payoff matrix to intervention is given by

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fissionable material and a local stock of nuclear weapons provides a ready supply.

Table 6: One Player Game

	medium k	weak k	none
medium j	$w-2c$		
weak j	$x-1-c$	$x-2$	
none	$-c$	$x-1$	0

Here is is clear that the only viable possibility are to not intervene and get 0, to intervene weakly on one sides and get $x-1$ or to make a medium intervention on both sides and get $w-2c$: which alternative is best depends on the parameters. If $x<1$ a weak intervention is too costly to be worthwhile; if w is large and c so that a bloody conflict is highly desirable and not much cost is incurred in supporting both sides then a medium intervention on both sides and a prolonged conflict will result.

The Great Game

We now explicitly consider two symmetric intervenors in competition with one another. We continue to assume no strong intervention and we retain the cost structure of $0, 1, c$ for no, weak and medium intervention. We assume that if neither side has an advantage both get zero, but each intervenor prefers their side to win getting x for a favorable hot peace and $h>x$ for a hegemony, and correspondingly getting losing that amount if the other side wins. With this we have the payoff matrix

Table 7: The Great Game

	medium k	weak k	none
medium j	$-c*, -c^*$	$x-c, -x-1$	$h-c*, -h$
weak j	$-x-1, x-c$	$-1, -1$	$-x-1, x^*$
none	$-h, h-c*$	$x*, -x-1$	0,0

If $h<c$ then none is dominant so let us rule out this uninteresting case. In the matrix above best responses are marked with asterisks: in this case we see that there is a unique Nash equilibrium with medium intervention on both sides and a prolonged conflict.

Prolonged War?

The model indicates two reasons for prolonged war. One is the great game between two competitors who want their side to win. The other is a single intervenor who wishes to weaken a pair of rival powers. Consider the locations of the historical instances of prolonged war brought about through outside intervention: Vietnam, Angola, El Salvador, Sudan, Lebanon, and Syria. The first three are cold war conflicts with the intervenors the US and USSR: the great game if ever there was

one. The final three are in the Muslim world. It is hard to believe that any outside non-Muslim power cares whether Sunni's or Shia's gain a hegemony - hence the cynical collusion theory has some force. If you believe - as many do - that a unified Muslim world in control of a substantial fraction of the world oil supply is a threat then keeping them fighting is worth something. Although to be clear from the Western European point of view the costs due to the (probably unanticipated) flow of refugees have been high. Although one might also cynically note that from the German perspective perhaps not as they have taken advantage of the refugee flow to remedy their difficult demographic situation. What is hard to believe in any of these conflicts that the behavior of the intervenors was motivated by moral considerations of peace and prosperity for the unfortunate inhabitants of these areas. In the case of the cold war such rhetoric was never used, and while rhetorically it has been used in the Sudan, Lebanon and Syria it is hard to reconcile these words with the behavior of the intervening powers.

8. Conclusion

The model we have studied sheds some light on the issues related to intervention and peace, in particular on the trade-off between having the contenders reaching peace as quickly as possible - which usually happens with the strong dominating the weak - and protecting the weak - which may prolong the conflict. We have further seen that if the goal of protecting the weak is predominant, then to minimize the costs of war intervention should be strong enough to avoid going back and forth between states where one part in turn is considerably stronger than the other, and reduce the war to what we have called a "hot peace" - which can be thought of as "border skirmishes", and hopefully end in reaching an unarmed negotiation stage.

However, much is left to understand. The great success story of peace is the de facto US occupation of Western Europe after World War II, its role in NATO and promoting the European Union and encouragement of European politicians, especially in France and Britain, that led to durable peace and democratic institutions. This was enormously costly, and this kind of peace - real peace - has lacked success elsewhere. Indeed US efforts at nation-building outside Western Europe and Japan has been an abysmal failure. The greatest success, ending the Israel-Arab conflict, has succeeded only in creating a hot peace propped up by continued and costly US intervention.

An important question is to understand why the U.S. was so successful in Europe and Japan and so unsuccessful elsewhere. Was it simply the willingness to commit resources on a massive scale - a huge war effort, military occupation on a giant scale, money poured into reconstruction? One may say it is the cold war - the willingness to actively support Europe to counteract the Soviets - but the cold war effort to intervene in Vietnam was as colossal a failure as the European intervention was a success. Certainly the success in Europe and Japan and failure elsewhere is something that needs to be understood. If it is simply a matter of resources and willingness to spend them, then perhaps the US success in Europe holds no useful lesson for peace.

In a similar vein we may wish to study earlier successful and unsuccessful attempts at nation building: for example, the British legacy in India is a stable and relatively peaceful democracy. The

French legacy in their colonies is poor - and the Belgian horrific. To understand success and failure here is to understand whether or not hegemony is a good idea. From the analysis here, however, it is reasonable to conclude that intervention to prevent hegemony needs to be strong enough (or weak enough) to bring about a hot peace - intervention that brings about prolonged war cannot be good from the point of view of peace.

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Appendix 1: Balance of Power Segments

We refer to balance of power segments of length one as short and balance of power segments of length greater than one as long.

Lemma 1. *In a j,k balance of power segment there are no states below the intervention thresholds, that is if $L_j(z) < \overline{L}_j$, or $L_k(z) < \overline{L}_k$, then z is not in a j,k balance of power segment.*

Proof. Consider j. Suppose first that $r_j^0(\phi_j(L_j(z)), \phi_k(L - L_j(z))) > 0$. Then there is zero resistance to j increasing land, but positive resistance to j decreasing land. Hence if $L_j(z)$ is in a segment it could only be a left endpoint. If this is the case then at z' where j has one more unit of land there would have to be zero resistance to j losing a unit of land. This would imply that at $L_j(z)$ there is outside help, that is $L_j(z) = \overline{L}_j$ - hence that in the segment $L_j \geq \overline{L}_j$.

Next suppose that $r_j^0(\phi_j(L_j(z)), \phi_k(L - L_j(z))) = 0$. Then if z was part of a balance of power segment it should be $L_j(z) > \overline{L}_j$, where j has positive resistance to losing land. But then it cannot be $L_j(z) < \overline{L}_j$ because below $\overline{L}_j$ resistance of j decreases with its land. □

Lemma 2. *Strong and ineffective intervention on behalf of j against k are ineffective in the sense that the balance of power segments remain unchanged if we take $\overline{L}_j = 0$.*

Proof. In the case of ineffective intervention the resistances are the same with or without intervention (recall that at $L_j > \overline{L}_j$ there is no intervention) so the balance of power segments are the same.

The effect of strong intervention is only to slow down the fall of resistance as j loses land, and it must fall to zero in any case because k's hegemonic resistance is positive even with intervention in favor of j. □

Lemma 3. *There is a paired balance of power segment if and only if intervention is weak for both j,k in which case the balance of power segments are the two short segments.*

Proof. Suppose intervention is weak for both j and k. The two segments in the assertion, namely from $\overline{L}_j$ to $\overline{L}_j + 1$ going to the right and from $\overline{L}_k$ to $\overline{L}_k + 1$ going to the left form indeed a paired segment, directly from the definitions. From Lemma 1 we know that no state to the left of $\overline{L}_j$ or to the right of $\overline{L}_k$, so what we have to show is that states between $\overline{L}_j + 1$ and $\overline{L}_k + 1$ do not belong to segments ("between" refers as usual to our preferred visualization). By hypothesis $r_k = 0$ at $L_k = \overline{L}_k + 1$, and it will be zero up to some $\overline{L}_k + \ell_k$; analogously, $r_j = 0$ from $\overline{L}_j + 1$ up to some $\overline{L}_j + \ell_j$ (going left in our pictures). If $\overline{L}_k + \ell_k$ and $\overline{L}_j + \ell_j$ do not overlap they must be one unit of land apart (formally, if $\overline{L}_k + \ell_k + \overline{L}_j + \ell_j < L$ their sum must be exactly $L - 1$), because at the state where one resistance becomes positive the other must be zero; thus from $\overline{L}_j + \ell_j$ down to $\overline{L}_j + 1$ there is zero resistance, and from $\overline{L}_j + \ell_j$ up-left to $\overline{L}_k + 1$ too; and in the "hole" between two adjacent states there is positive resistance both ways. Therefore in this case there is no segment between the two segments. Suppose now that If $\overline{L}_k + \ell_k$ and $\overline{L}_j + \ell_j$ do overlap. The only difference from the previous case is that instead of the hole there is an interval between the two segments

where points are linked by two-way zero-resistance; from the left end point of the interval there is zero resistance down to $\overline{L}_j + 1$, and from the right and point analogously up to $\overline{L}_k + 1$. None of these states can belong to a segment, as before.

Conversely, suppose there is a paired balance of power segment. Let us call "walls" the extremes of the segments. At the left wall of the rightmost segment j's resistance is positive, then going down, at the right wall of the leftmost segment j's resistance is zero (since k has positive resistance); and at the left wall of the left segment j's resistance is positive again. Therefore here is where support for j occurs, that is the left wall of the left segment is $\overline{L}_j$. Similarly the right wall of the right segment is $\overline{L}_k$. Starting from there going down to the left wall of the same segment there is no support for j but still its resistance is positive there. If the right segment were not short, in its interior j's resistance would have to be zero, but it is again positive at the left wall, which is impossible given there is no external support in that range. Thus the right segment is indeed the short one in the assertion, which implies intervention in support of k is weak as asserted. Notice in passing that j's resistance would be positive at the right wall if it were not for intervention in favor of k. The argument for the left segment is the same. □

Lemma 4. *If intervention for j is weak and that for k is medium or there is no intervention for k there is a single short balance of power segment starting at $\overline{L}_j$ and the intervention on behalf of k is ineffective.*

Proof. Existence of a short segment starting at $\overline{L}_j$ follows as in Lemma 3 from weakness of intervention for j: at the threshold j has positive resistance (since the intervention is not very weak) and so k has zero, we can move only to the right; but when we move one unit to the right, by definition of weak intervention k has positive resistance, hence j has zero and we can move only to the left; hence as we can move back and forth only between $\overline{L}_j$ and $\overline{L}_j + 1$ we have a short segment. Since we know (Lemma 1) that states on the left of $\overline{L}_j$ cannot be part of a segment, to establish uniqueness we only need to look to the right $\overline{L}_j + 1$.

Suppose there is no intervention for k. At $\overline{L}_j + 1$ there is positive resistance by k, and increasing j's land, as long as this is the case there is zero resistance to going left one step; at some point k's resistance may become zero (so there is a hole there) and from then on to j's hegemony k's resistance must remain null. Thus we have no other segment.

Suppose now that intervention for k is strong. Then it must be the case that $\overline{L}_k$ is to the right of $\overline{L}_j + 1$, for otherwise j's resistance at $\overline{L}_j$ would be zero. From $\overline{L}_j + 1$ up to where $L_k = \overline{L}_k$ we must have null resistance by j - since it is zero where $L_k = \overline{L}_k$, going left j has less land and no support in the range. And to the right of $\overline{L}_k$the situation is essentially as in the previous case: as we increase j's land, the system can move one step to the left without resistance until a possible "hole" after which j's resistance becomes positive, then right to j's hegemony without resistance. Again there is no segment in the range.

As we have just seen the two cases yield the same stable configuration, so that intervention for k is ineffective. □

Lemma 5. *If intervention for j is medium and there is no intervention for k there is no recurrent balance of power segment.*

Proof. By Lemma 1 no state below $\overline{L}_j$ can belong to a segment; at $L_j = \overline{L}_j + 1$ resistance by k is zero, and absent intervention in its favor k's resistance remains null until j reaches hegemony, by monotonicity. Conclusion follows. □

Lemma 6. *If intervention for both is medium then there is a single long recurrent balance of power segment from $\overline{L}_j$ to $L - \overline{L}_k$.*

Proof. Again by Lemma 1 states with $L_j < \overline{L}_j$ or $L_k < \overline{L}_k$ cannot be part of a segment. By hypothesis at $L_j = \overline{L}_j + 1$ resistance by k is zero, hence it remains zero up to where $L_k = \overline{L}_k + 1$, then becomes positive at $\overline{L}_k$ (where necessarily j's resistance is null). Similarly, going left from $\overline{L}_k$ by hypothesis j's resistance is zero where $L_k = \overline{L}_k + 1$ and by monotonicity it remains zero until where $L_j = \overline{L}_j + 1$, then it is positive at $L_j = \overline{L}_j$ (where k's resistance is null). This is what we had to show. □

Appendix 2: Stochastic Stability of Hegemony and Balance of Power

Theorem. *[Theorem 3 in the text] There exist $\infty \geq \overline{\varphi}_0 \geq \underline{\varphi}_0 > 0$ such that if $\varphi_0 > \overline{\varphi}_0$ and if intervention thresholds are positive on both sides there are stochastically stable balance of power segments but not stochastically stable hegemonies, while if $\varphi_0 < \underline{\varphi}_0$ there are stochastically stable hegemonies but not stochastically stable - or even absorbing - balance of power segments.*

Proof. Hegemonic resistance is decreasing in φ_0 and by assumption positive for $\varphi_0 = 0$. At $\varphi_0 = 0$ there are no balance of power segments with positive radius and by continuity this is the case for all sufficiently small φ_0. Hence there is $\underline{\varphi}_0$ below which there are stochastically stable hegemonies but not balance of power segments.

By Assumption 1 hegemonic resistance by assumption falls to zero for all sufficiently large φ_0 while for sufficiently large φ_0 the resistance to passing an intervention threshold is positive. Hence there is a $\overline{\varphi}_0$ above which there are stochastically stable balance of power segments but not hegemonies. □

Appendix 3: Dynamics and Modified Radii

How long, relatively speaking, will the system spend in the absorbing sets we have encountered, for small ϵ? The analysis of the dynamics of the system hinges on two numbers describing an absorbing sets containing a point z: the *radius* R_z which represents roughly speaking the resistance to escaping "far" from that state; and the *modified radius* $M_z \geq R_z$, which is a broader measure but the same idea. The dynamics (for $\epsilon > 0$) can be well described by these two numbers: we have an extension of the results concerning the relative ergodic resistance for hegemonies in the simple model without outside intervention. As before, the expected length of time before leaving

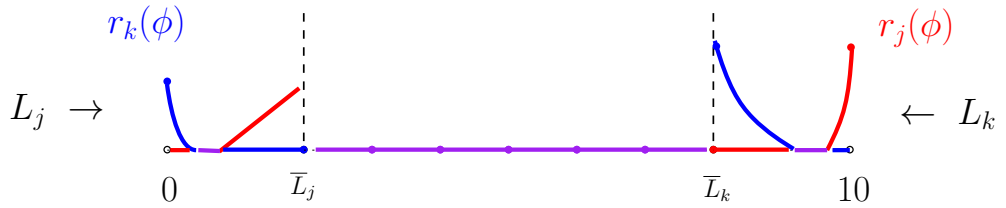
an absorbing configuration to reach another is of order ϵ^{-R_z}. Configurations with $R_z = 0$ also have $M_z = 0$ and are not absorbing. In particular the stable sets are those with maximum modified radius.

The radius of an absorbing set is the least resistance needed to reach a different absorbing set, and its computation in our setting is always simple. We already saw a special case of this in the model without outside intervention, where the radius of an hegemony is the resistance to reach the only other absorbing set of the system, that is the other hegemony. The modified radius embodies the notion of the resistance needed to leave the set, but takes account of what path the system might take after leaving.

As we indicated, the radius governs the length of time it takes to leave an absorbing set. Specifically if we let $T_\epsilon(z)$ denote the expected hitting time before reaching a different absorbing set starting at z then $T_\epsilon(z)$ is of order ϵ^{-R_z}. By contrast the expected length of time it takes to actually make the leap from one absorbing set to another is much shorter: it is bounded independent of ϵ. If we are interested in the relative amount of time spent in two absorbing sets containing the points z and z' respectively then the answer is given by the modified radius: the ratio of $\mu_\epsilon(z)/\mu_\epsilon(z')$ is of order $\epsilon^{M_{z'}-M_z}$. This is proven in Levine and Modica (2016).

We refer to balance of power segments of length one as short and balance of power segments of length greater than one as long. Consider for an illustration the case of an absorbing single long segment, as in figure 8.1. Here, as always, L_j is measured from left to right and L_k from right to left and the arrows in the figure denote the zero-resistance transitions. The radius is the least resistance to reach an hegemony. Hence we compute a *left radius* R^ℓ_{jk} which is the total resistance to reach the left hegemony, a *right radius* similarly for the right hegemony, and conclude that for all z in the segment the radius is $R_z = \min\{R^\ell_{jk}, R^r_{jk}\}$. To wit, the left radius is computed as follows: start at the left endpoint $\overline{L}_j$; reduce the land holding of j one unit at a time, that is from $\overline{L}_j$ to $\overline{L}_j - 1$ and so on. Each time compute the resistance to the land loss (we know from Theorem 2 that since this is absorbing the first step at least has positive resistance); add these numbers together and continue until j has lost all their land and become inactive. That is, we take $r_j(\phi_j(\overline{L}_{jk}), \phi_k(L - \overline{L}_{jk}))$, add to it $r_j(\phi_j(\overline{L}_{jk} - 1), \phi_k(L - \overline{L}_{jk} + 1))$ and continue until all land is lost. This gives R^ℓ_{jk}. Similarly we compute R^r_{jk} going rightward.

Figure 8.1: Single long balance of power segment



The long segment is between $L_j = \overline{L}_j$ and $L_j = L - \overline{L}_k$ (indicated in the figure by the label $\overline{L}_k$ using the reverse axis). j's resistance r_j jumps up at $\overline{L}_j$ because of intervention.

In this appendix we compute the modified radii of hegemonies and balance of power segments.

The attribute of an absorbing set that determines the relative time the process spends on it is the *modified radius*, which we are going to compute for the absorbing sets we are interested in here. The general definition is given in Levine and Modica (2016), section 6.3, we sketch here the concept to apply it to the present setting. We say that a family of absorbing sets forms a *circuit* if any two of them are connected by a least resistance path not touching absorbing sets outside the family. The family of all absorbing sets of the process can be partitioned into circuits, and then one defines circuits of circuits - 2nd-order circuits we may say - by taking as modified resistance the incremental resistance needed to move from one to the other over that needed to move within the circuit. Continuing this way one gets to 3rd-order circuits and so on (modified resistance becomes accordingly increments of increments etc.) until all circuits of order $K-1$ form a single circuit of order K. The modified radius of an absorbing set is computed as the least total modified resistance needed to move out of the k-order circuits in which the absorbing set is contained, all the way up to the order $K-1$. We now apply this procedure starting with a particularly simple case.

The case of rightward cascades

We say that a path between two absorbing sets is *direct* if it does not touch other absorbing sets along the way and has finite resistance. A *rightward cascade* $x_1 \to x_2 \ldots \to x_n$ is a sequence of absorbing sets such that the only direct paths from x_i to other absorbing sets lead to the adjacent x_j's. Denote the resistance of going back to x_{i-1} by R_k^l and of going forward by R_i^r (the left and right radii). To be a rightward cascade we require that $R_i^r < R_i^l$ for $i \neq 1, n$, and we define the difference $\Delta_i = r_{il} - r_{ir}$ which is by assumption strictly positive. In this case the hierarchy of circuits and the modified radii are as follows. The only non singleton circuit is given by $\{x_{n-1}, x_n\}$; then one order up we find the 2-nd order circuit $\{x_{n-2}, \{x_{n-1}, x_n\}\}$, all the others being singletons. Proceeding this way we arrive at the two-element partition $\{x_1, \{x_2, \{x_3\{\ldots\{x_{n-1}, x_n\}\}\ldots\}$ after which we have a single circuit. So from x_1 we only need to move right to x_2, while from all the others we need to move up to x_1. For example for $n = 4$ we have $\{x_1, \{x_2, \{x_3, x_4\}\}\}$; to get out from x_1 we only need R_1^r; from x_2 we need R_2^l to move out to x_1 (notice that this is incremental as well since there is zero resistance to move within the singleton circuit formed by x_2 alone); from x_3 we need R_3^l to step out of the circuit to x_2 and then the incremental resistance of moving from x_2 to x_1 which is Δ_2 (for the resistance of moving from x_2 within this 2nd-order circuit circuit is R_2^r), so the modified radius is $R_3^l + \Delta_2$; from x_4 we need to first reach x_3 to get out of the circuit plus the increments up to x_1, so the modified radius is $R_4^l + \Delta_3 + \Delta_2$. The general rule is then as stated in the following

Proposition 1. *The modified radius of x_i, denoted by M_i, is given by $M_i = R_i^l + \sum_{1<k<i} \Delta_k$ for $i > 1$, while $M_1 = R_1^r$.*

Cascade with One Balance of Power Segment

It is convenient to denote the hegemony of k by H that of j by h and the balance of power segment by B in the following. Notice that in this case we must have a cascade and we may assume

without loss of generality the cascade is to the right $H \to B \to h$. Apply Proposition 1 to find, $M_H = R_H^r$, $M_B = R_B^l$, $M_h = R_h^l + (R_B^l - R_B^r)$.

Two Balance of Power Segments

case 1: Cascade

Let B denote the segment on the left and b the one on the right. Assume without loss of generality the cascade is to the right: $H \to B \to b \to h$. Apply Proposition 1 to find $M_h = R_h^l + (R_b^l - R_b^r) + (R_B^l - R_B^r)$, $M_b = R_b^l + (R_B^l - R_B^r)$, $M_B = R_B^l$, $M_H = R_H^r$.

Comments. We have $M_b = M_B + R_b^l - R_B^r$; if $R_b^l > R_B^r$ as we may suppose, then $M_b > M_B$. Also $M_h = R_h^l - R_b^r + M_B + R_b^l - R_B^r = R_h^l - R_b^r + M_b$; hence, assuming $R_b^l > R_B^r$ that is $M_b > M_B$, then $M_h \gtrless M_b \iff R_h^l \gtrless R_b^r$. So if $R_h^l > R_b^r$ the stable set is a hegemony. Otherwise b is stable if R_H^r is low enough.

case 2: *Two Circuits*

Then both balance of power segments point at the adjoining hegemony and the circuits are $\{H, B\}, \{b, h\}$. At the higher order both form a single circuit. The incremental resistances to reach the other circuit are $R_B^r - R_B^l$ and $R_b^l - R_b^r$ so the modified radii are $M_H = R_H^r + R_B^r - R_B^l$, $M_B = R_B^r$, $M_b = R_b^l$, $M_h = R_h^l + R_b^l - R_b^r$.

case 3: *Balance of Power Segments Point at Each Other - Three Circuits* $H, \{B, b\}, h$

In the circuit the radii are R_B^r, R_b^l and the incremental resistance of going to H is $R_B^l - R_B^r$, while that of going to h is $R_b^r - R_b^l$. Suppose without loss of generality $R_b^r - R_b^l < R_B^l - R_B^r$ so that the 2nd-order circuit is $\{\{B, b\}, h\}$. The modified radius of H is just $M_H = R_H^r$. For B there is the resistance of going to h which is $R_B^r + \left(R_b^r - R_b^l\right)$ - path to b then incremental resistance to reach h; then the modified resistance of going from this to H, which is the difference between incremental resistance of going to H and to h (incremental with respect to remaining in $\{B, b\}$, that is $R_B^l - R_B^r - (R_b^r - R_b^l)$. So the modified radius of B is $M_B = R_B^r + \left(R_b^r - R_b^l\right) + \left(R_B^l - R_B^r - (R_b^r - R_b^l)\right) = R_B^l$. Since the difference of the modified radius of b with the modified radius of B must be $R_b^l - R_B^r$ both being in the same circuit we have $M_b = R_b^l + (R_B^l - R_B^r)$. For h: we must first get to $\{B, b\}$ - which costs R_h^l; then there is the step to H, whose modified resistance we have already calculated; thus $M_h = R_h^l + (R_B^l - R_B^r) - (R_b^r - R_b^l)$. An alternative, sleeker derivation of M_h: the modified radius is the cost of going to H from b, that is $R_b^l + (R_B^l - R_B^r)$, plus the extra cost of starting from h, which is the difference $R_h^l - R_b^r$ between the radii of h and b in the 2nd-order circuit $\{\{B, b\}, h\}$; the result is $M_h = R_b^l + (R_B^l - R_B^r) + R_h^l - R_b^r$ which is the same value as before.

Comments (assuming supercircuit $H, \{\{B, b\}, h\}$). Let us suppose $R_b^l \approx R_B^r$ so that $M_B \approx M_b$. Next, $M_H \gtrless M_B \iff R_H^r \gtrless R_B^l$; and $M_h = M_b + R_h^l - R_b^r$ so $M_h \gtrless M_b \iff R_h^l \gtrless R_b^r$. Stability is like in the single segment case: if hegemonic resistances are higher than resistance from segments to hegemony then a hegemony is stable; otherwise the stable set is a segment (or both).