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An Economic Test for an Unlawful Agreement to Adopt a Third-Party's Pricing Algorithm

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Abstract

AI has helped fuel a growing market in the supply of pricing algorithms by software developers. While there is an efficiency rationale for outsourcing pricing, anticompetitive concerns have been expressed when competitors in a market adopt the same pricing algorithm. These concerns have resulted in private litigation claiming a third-party company (who developed the pricing algorithm) and firms (who adopted it) had an unlawful agreement. This study develops an empirical test for determining whether firms' adoption decisions are coordinated. If adoption decisions are coordinated then adopters' average price is increasing in the number of adopting firms, while if adoption decisions are independent then adopters' average price does not depend on the number of adopting firms. This test could provide economic evidence to support a claim of an unlawful agreement between a third-party developer and adopting firms.

*I appreciate the comments of two anonymous referees, the editors, and participants at the Learning in Games and Algorithmic Collusion Workshop at the Oxford-Man Institute (October 2023). I have not been retained by any party in any of the current legal cases involving third-party pricing algorithms.

1 Introduction

Awash in data, firms face the challenge of figuring out how to most effectively use it to augment decision-making and do so in real time. AI is playing an increasing role in addressing that challenge whether it involves managing inventories, the supply chain, or customer relations (Haan, 2023). Access to the benefits of AI is not limited to large companies for an advantage of data-driven decision-making is that it can be outsourced. A firm can share its data with a third party who, with superior programming expertise, computing power, and data, is better equipped to use it to assist decision making in a cost-effective manner. Just like any "make or buy" decision, a firm may find it more efficient to buy the input - here it is support for decision-making - rather than do it themselves. That creates an opportunity for IT service providers to invest and innovate to attract more business customers. Leaving these third-party vendors unfettered in how they design their services and to whom they sell would seem to be surest way to realize the greatest benefits.

The matter is more complicated, however, when the service that an IT company is supplying is guidance on how a firm should price. Through its design and implementation of a pricing algorithm, a third party can make it possible for a firm to adjust its price more rapidly to high-frequency demand and supply shocks, which will increase transaction volume and social welfare. By tailoring price to more narrowly-defined market segments, a third party's pricing algorithm allows for more precise price discrimination which increases profits and benefits those consumers (who are typically lower income) who pay a lower price. These efficiencies are attractive but are also the source of a conundrum. An IT company who successfully innovates is rewarded with widespread adoption of its service which could mean competitors in a market are using a pricing algorithm supplied by the same third party. Given an input supplier will seek to create value for its customers - as doing so will allow it to charge more and sell more - an IT service provider with a customer base comprising competitors may be inclined to design the pricing algorithm to reduce competition and thereby raise adopters' profits. Concerns about these possible anticompetitive effects have been widely and regularly expressed:

OECD: "[C]oncerns of co-ordination could arise if firms outsourced the creation of algorithms to the same IT companies and programmers."¹

United Kingdom's Competition & Markets Authority: "If a sufficiently large proportion of an industry uses a single algorithm to set prices, this could result in ... the ability and incentive to increase prices."²

Testimony before the U.S. Congress: "[Another] area of concern with the use of pricing algorithms seems more subtle and hard to detect: Companies avoiding price competition by using the same third-party vendor to collect data on supply and demand and 'recommend' pricing or output behaviors that facilitate price

¹OECD, "Algorithms and Collusion - Background Note by the Secretariat," DAF/COMP(2017)4, 9 June 2017, para. 68.

²United Kingdom Competition & Markets Authority, "Pricing Algorithms: Economic Working Paper on the Use of Algorithms to Facilitate Collusion and Personalised Pricing," 8 October 2018, para. 5.21.

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3 coordination.”³
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5 German Monopolies Commission: “It may happen that such an IT service
6 provider sells an algorithm that it knows or accepts could contribute to a collusive
7 market outcome. It is even conceivable that individual IT service providers see
8 such a contribution as an advantage, as it makes the algorithm more attractive
9 for users interested in profit maximization.”⁴
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12 There is some evidence, and claims of evidence, that these anticompetitive effects have
13 occurred. Software developer a2i Systems created a pricing algorithm to assist retail gasoline
14 companies in the pricing of their product. This is a natural market for a third-party developer
15 to enter because the complexity of pricing in this market is well documented (Eckert, 2013).
16 Gasoline pricing can involve high-frequency price changes - often several times over the
17 course of a day - and price cycles where prices sharply rise then gradually fall with that
18 pattern repeating itself. There is then likely to be room for a third party to develop a more
19 effective pricing rule than is currently being used. a2i Systems’s software was widely adopted
20 in Germany and a recent study by Assad, Clark, Ershov, and Xu (2023) found evidence of
21 anticompetitive effect: “Adoption increases margins, but only for non-monopoly stations.
22 In duopoly and triopoly markets, margins increase only if all stations adopt, suggesting
23 algorithmic pricing has a significant effect on competition.”⁵
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26 Though the claims are still to be evaluated by the judicial process, plaintiffs in several
27 legal cases in the United States argue that an IT service provider and competitors in a mar-
28 ket had an illegal agreement to restrain competition. In the market for apartment rentals,
29 software developers RealPage and Yardi designed and sold pricing algorithms to be used
30 by apartment property owners, and are now being sued in separate litigation for violating
31 Section 1 of the Sherman Act.⁶ According to RealPage, many owners have historically used
32 simple heuristics in setting rents. If that is the case, there is room for a third party to develop
33 a more profitable pricing rule. But plaintiffs claim that RealPage went farther as it delivered
34 value to adopters by reducing competition between them, and adopters were complicit.⁷ A
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38 ³Written Testimony of Bill Baer, U.S. Senate Committee on the Judiciary, Subcommitt-
39 tee on Competition Policy, Antitrust, and Consumer Rights, Hearings on “The New Invisible
40 Hand? The Impact of Algorithms on Competition and Consumer Rights,” December 13, 2023, p.
41 2. <https://www.judiciary.senate.gov/imo/media/doc/2023-12-13_pm_-_testimony_-_baer.pdf> (down-
42 loaded January 17, 2024) Bill Baer is a former Assistant Attorney General of the Antitrust Division of the
43 U.S. Department of Justice.

44 ⁴German Monopolies Commission, XXII. Biennial Report, Chapter on “Algorithms and Collusion,” 2018,
45 para. 263.

46 ⁵Abstract of Assad, Clark, Ershov, and Xu (2023).

47 ⁶The complaint against RealPage is *Bason, et al. v. RealPage, Inc., et al.*, No. 3:22-cv-01611-WQH-
48 MDD, U.S. District Court, Southern District of California, Oct. 18, 2022; and against Yardi is *Duffy, et al.*
49 *v. Yardi Systems, Inc., et al.*, No. 2:2023-cv-0139, U.S. District Court, Western District of Washington at
50 Seattle, Sept. 8, 2023. It has also been reported that “[t]he Department of Justice’s Antitrust Division has
51 opened an investigation into whether rent-setting software made by a Texas-based real estate tech company
52 is facilitating collusion among landlords, according to a source with knowledge of the matter.” (“Department
53 of Justice Opens Investigation Into Real Estate Tech Company Accused of Collusion with Landlords,” by
54 Heather Vogell, ProPublica, November 23, 2022 [https://www.propublica.org/article/yieldstar-realpage-rent-
55 doj-investigation-antitrust](https://www.propublica.org/article/yieldstar-realpage-rent-doj-investigation-antitrust))

56 ⁷“RealPage and participating Lessors have provided one another with such mutual assurances, agreeing
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3 recent study by Calder-Wang and Kim (2023) offers evidence for both procompetitive and
4 anticompetitive effects: “Our findings suggest that algorithm adoption helps building man-
5 agers set more responsive prices: buildings with the software increase prices during booms
6 but lower prices during busts, compared to non-adopters in the same market. However, we
7 also find evidence that greater algorithm penetration can lead to higher prices, raising rents
8 among both adopters and non-adopters in the same market.”⁸

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10 In the market for hotel rooms, third-party developer Rainmaker developed a pricing al-
11 gorithm to assist hoteliers in pricing more profitably.⁹ As with retail gasoline and apartment
12 rentals, there is a plausible efficiency basis, for hotel demand is highly dynamic as it is sub-
13 ject to demand variation (e.g., seasonal fluctuations and demand-increasing events such as
14 conventions) and hotel supply is a perishable good (e.g., a hotel room for August 1, 2023
15 is lost forever after that date). Consequently, profit-maximizing prices change as a date
16 approaches and do so in a potentially complex way. There is then a legitimate basis for
17 a software developer to invest in creating a superior pricing rule and for firms to adopt it.
18 Nevertheless, there may be an incentive for it to design a collusive pricing algorithm and to
19 facilitate an agreement among competitors to adopt it, which is what is being claimed by
20 plaintiffs who bought a hotel room in Las Vegas or Atlantic City during the time that some
21 hotels were using Rainmaker’s pricing algorithm.¹⁰

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23 In light of what is going on in the markets for apartments, gasoline, and hotels, the
24 antitrust problem of competitors adopting a third-party developer’s pricing algorithm is
25 here and now. From a welfare perspective, how do we determine when competitors utilizing
26 the same pricing algorithm provided by a third party is harmful to consumers? How do we
27 permit adoption on efficiency grounds while preventing it when it is anticompetitive? Unless
28 one is prepared to prohibit firms in the same market from adopting a pricing algorithm
29 from the same third party - which is extreme and unjustified by the existing evidence - we
30 face the challenge of distinguishing between adoptions that are anticompetitive and those
31 that are not. From a legal perspective, how do we determine when competitors adopting a
32 third-party developer’s pricing algorithm is a violation of competition law? How do we know
33 when competitors’ adoption decisions were coordinated not independent, and the third-party
34 developer facilitated those joint adoption decisions?

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36 This study addresses the challenge of determining when a third party (who developed

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41 among themselves not to compete on price for the sale of multifamily residential real estate leases. They
42 have effectuated their agreement through two mutually reinforcing mechanisms. First, participating Lessors
43 have agreed to set prices using RealPage’s coordinated algorithmic pricing. Second, participating Lessors
44 have agreed to stagger their lease renewal dates through RealPage, to avoid (otherwise natural) oversupplies
45 in rental properties.” *Bason, et al. v. RealPage, Inc., et al.*, para. 46.

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47 ⁸Abstract of Calder-Wang and Kim (2023).

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49 ⁹For the Las Vegas market, the complaint is *Richard Gibson, et al. v. MGM Resorts International, et al.*, No. 2:23-cv-00140, U.S. District Court, District of Nevada, January 25, 2023; and for the Atlantic City
50 market, the complaint is *Altman, et al. v. Caesars Entertainment, Inc., et al.*, No. 2 :23-cv-02536, U.S.
51 District Court, District of New Jersey, May 9, 2023.

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53 ¹⁰“In furtherance of the contract, combination, or conspiracy, Defendants have committed one or more
54 of the following acts: a) provided information to be used in the operation of the pricing algorithms; b)
55 created and operated algorithms that provided pricing recommendations to Defendants; c) knowingly used
56 algorithms that incorporated information from other Defendants in setting pricing recommendations; and/or
57 d) set prices based in whole or in part on pricing recommendations provided by Rainmaker Group.” *Richard
58 Gibson, et al. v. MGM Resorts International, et al.*, para. 89.

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3 the pricing algorithm) and competitors in a market (who adopted it) have an agreement
4 in violation of competition law. Obviously, evidence of documented communications to
5 coordinate their conduct would be most convincing to the courts. This could involve direct
6 and express communications between competitors and the third party or, as with a hub-and-
7 spoke cartel, the third party using bilateral communications with competitors in order to
8 establish a common understanding to adopt the pricing algorithm. However, let us suppose
9 evidence of communications is insufficient, perhaps because it is not express or is not private
10 (such as a third-party developer engaging in public announcements which could facilitate
11 coordination though argued by them to be nothing more than marketing).¹¹ In that situation,
12 circumstantial evidence is required.¹² One class of circumstantial evidence is referred to as
13 "plus factors" in the United States and the concept is relevant to other jurisdictions.¹³ One
14 of the more compelling plus factors is conduct against a firm's independent self-interest.
15 Examples are a firm adopting a higher price or choosing to no longer negotiate price with
16 customers. It is not difficult to argue that raising prices or prohibiting discounts off of list
17 prices would be unprofitable unless rival firms also chose to do so. However, the argument
18 is more challenging with the adoption of a third-party developer's pricing algorithm because
19 it can be profitable to adopt even if rival firms do not. If the pricing algorithm allows a
20 firm to more effectively respond to demand shocks or engage in price discrimination, that
21 can enhance profit while holding fixed rival firms' pricing rules. It is then more challenging
22 to use this plus factor when the claimed agreement is adopting a third-party developer's
23 pricing algorithm because the pricing algorithm may enhance efficiency as well as restrain
24 competition.

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30 The contribution of this study is to offer a plus factor for proving an unlawful agreement
31 between the third party who developed the pricing algorithm and competitors in a market
32 who adopted it.¹⁴ If a third-party developer is part of an agreement with competitors, it will
33 then design the pricing algorithm differently than if there is no agreement. More specifically,
34 if adoptions are coordinated then adopters' prices will be increasing in the adoption rate
35 (i.e., the fraction of firms who adopt) and, on average, adopters will price higher than non-
36 adopters. In contrast, if firms' adoption decisions are independent then adopters' prices do
37 not change with the adoption rate and, on average, adopters and non-adopters price the
38 same. While the prediction under coordinated adoptions is expected - for the third party
39 is effectively acting as a cartel manager - the prediction under independent adoptions is
40 not. Even with independent adoptions, one might think that a third party would reduce
41 competition in order to create more value from adoption; however, I show that is not the
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45 ¹¹As reviewed in Harrington (2022a), U.S. courts have been reluctant to find an unlawful agreement based
46 only on public announcements, even egregious ones.

47 ¹²"Circumstantial evidence can come in several forms, including evidence of communications between
48 rivals and economic evidence. Economic evidence consists of firm conduct, market structure, and evidence of
49 facilitating practices. All types of evidence can be useful in a case and they should be employed together."
50 OECD, "Prosecuting Cartels without Direct Evidence" DAF/COMP/G(2006)7, 11 September 2006, p. 18.

51 ¹³"One formulation, developed in the United States in civil cases ..., requires that there exist certain 'plus
52 factors,' which prove that agreement is more likely the cause of the parallel conduct than independent action.
53 ... Other jurisdictions do not use the same terminology as US courts, but it seems that the analysis that
54 they apply is similar." OECD, "Prosecuting Cartels without Direct Evidence" DAF/COMP/G(2006)7, 11
55 September 2006, p. 29.

56 ¹⁴Identifying plus factors for collusion involving algorithms is discussed in Gal (2019).

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3 case. Higher average prices result only when adoption decisions are coordinated. Therefore,
4 the properties of the pricing algorithm can inform us when the pricing algorithm is designed
5 for firms who have an agreement to adopt it.
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7 In considering the body of research pertaining to algorithms and collusion, there is an
8 extensive and growing theoretical literature on learning algorithms resulting in supracompet-
9 itive prices.¹⁵ While this practice has not yet been documented in actual markets, it would
10 create a serious antitrust challenge should it occur.¹⁶ That situation is to be contrasted with
11 when a third party develops the pricing algorithm. The fundamental economic difference is
12 that a firm designing its own pricing algorithm does so in order to maximize its profit from
13 *using* it, while a third-party developer designs a pricing algorithm in order to maximize its
14 profit from *selling* it. A crucial implication is that a third-party developer will recognize that
15 its pricing algorithm will "compete against itself" when adopted by competitors in a market.
16 What this means for the design of the pricing algorithm and possible anticompetitive conduct
17 is largely not understood, for the literature on the provision of pricing algorithms by a third
18 party is scarce. As mentioned above, there are two empirical studies: Assad et al (2023) and
19 Calder-Wang and Kim (2023). As regards theoretical analysis, there is only previous work
20 by the author (Harrington, 2022b). The analysis of the current paper extends Harrington
21 (2022b) from the case of two firms to when there are many firms so that predictions can be
22 derived regarding how prices depend on how many firms adopt.¹⁷
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25 Section 2 describes the model along with some preliminary analysis. In Section 3, a
26 third-party developer's pricing algorithm is characterized when firms' adoption decisions are
27 coordinated and when they are independent. Section 4 compares the pricing algorithm under
28 coordinated and independent adoptions. A description of the plus factor for establishing
29 an unlawful agreement is provided in Section 5 along with a discussion of its empirical
30 implementation. Section 6 concludes.
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34 35 2 Model

36 37 2.1 Model and Equilibrium without a Third-Party Developer

38 Consider a market where firms have symmetrically differentiated products and a common
39 and constant marginal cost c . For tractability purposes, demand is assumed to be linear:
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43 ¹⁵Work on algorithmic collusion (either adopting supracompetitive prices or collusive pricing rules) includes
44 Waltman and Kaymak (2008), Kimbrough and Murphy (2009), Cooper, Homen-de-Mello, and Kleywegt
45 (2015), Calvano, Calzolari, Denicolò, and Pastorello (2020, 2021), Eschenbaum, Mellgren, and Zahn (2021),
46 Hansen, Misra, and Pai (2021), Kastius and Schlosser (2021), Klein (2021), Musolf (2021), Abada, Lambin,
47 and Tchakarov (2022), Banchio and Mantegazza (2022), van den Boer, Meylahn, and Schinkel (2022), Cartea,
48 Chang, Penalva, and Waldon (2022), Leisten (2022), Meylahn and den Boer (2022), Sanchez-Cartas and
49 Katsamakas (2022), Abada and Lambin (2023), Asker, Fershtman, and Pakes (2023), Brown and MacKay
50 (2023), Epivent and Lambin (2023), and Possnig (2023).

51 ¹⁶For a discussion of the legal challenges, see Mehra (2016), Ezrachi and Stucke (2017), Johnson (2017),
52 Oxera (2017), Deng (2018), Harrington (2018), Gal (2019), Schwalbe (2019), Calvano, Calzolari, Denicolò,
53 Harrington, and Pastorello (2020), Beneke and Mackenrodt (2021), Asil and Wollmann (2023), Ezrachi and
54 Stucke (2023), and Gal (2023).

55 ¹⁷A distinct but related set of issues arises with competitors who contract a third party to submit bids on
56 their behalf at online advertising auctions; see Decarolis, Goldmanis, and Penta (2020).
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$a - bp_i + dP_{-i}$ where p_i is a firm's own price, P_{-i} is the average price of rival firms, and $b > d > 0$. So as to allow for multiple firms in a tractable manner, there is assumed to be a continuum of firms.¹⁸ A critical feature is demand variation with respect to a which is assumed to have a continuously differentiable cdf $G : [\underline{a}, \bar{a}] \rightarrow [0, 1]$ with mean μ and variance σ^2 . The higher is a , the stronger is firm demand in the sense that, for any price vector, the amount of demand is larger and demand is more price-inelastic. Assume $\underline{a} - (b - d)c > 0$ so, at the static Nash equilibrium, demand is positive for all demand states. The demand variable a has two interpretations. A firm could be facing a single demand curve and a is a demand shock with distribution G . In that case, price may condition on the current demand shock. Alternatively, a firm faces a collection of market segments represented by G . In that case, price may condition on the market segment a .

In the absence of a third party, it is supposed that the demand shock a occurs at a higher frequency than a firm's pricing decisions or, when G represents a collection of market segments, the firm cannot distinguish among those segments when pricing. In that case, a firm is incapable of conditioning price on the demand state. A symmetric Nash equilibrium price p^N is defined by:

$$p^N \equiv \arg \max_{p_i \in \mathbb{R}_+} \int (p_i - c) (a - bp_i + dp^N) G'(a) da \Leftrightarrow p^N = \frac{\mu + bc}{2b - d}.$$

The associated expected profit is

$$\pi^N \equiv \int \left(\frac{\mu + bc}{2b - d} - c \right) \left(a - (b - d) \left(\frac{\mu + bc}{2b - d} \right) \right) G'(a) da = \frac{b(\mu - (b - d)c)^2}{(2b - d)^2}.$$

A useful benchmark for subsequent analysis is when a fraction θ of firms can condition price on the demand state and the remaining firms cannot and thus set a uniform price. It is shown in the Appendix that the pricing rule for the firms who can condition price on the demand state is¹⁹

$$p^F(a) \equiv \frac{d(1 - \theta)\mu + (2b - d\theta)bc}{(2b - d\theta)(2b - d)} + \left(\frac{1}{2b - d\theta} \right) a, \quad (1)$$

and the firms who set a uniform price choose:

$$\frac{\mu + bc}{2b - d}.$$

Note that

$$E_a [p^F(a)] = \frac{\mu + bc}{2b - d}$$

so average price is the same for all firms.

¹⁸This demand specification is the extension of a common specification - see, for example, Vives (1999), p. 146 - to when there is a continuum of firms. The advantage of a continuum of firms is that it simplifies the analysis because, as we will see, an individual firm's adoption decision does not affect the third party's pricing algorithm. That property is a good approximation for when there are many firms in the market such as many small apartment property owners.

¹⁹The superscript F refers to the pricing algorithm being designed by the f(irm) rather than the third party.

2.2 Model with a Third-Party Developer

Let us now introduce a single third-party developer whose business model is to design and sell pricing algorithms. The efficiency delivered by the third-party developer is that its pricing algorithm can track the high-frequency demand shock (or market segment) a and condition price on it. An algorithm is denoted $\phi : [\underline{a}, \bar{a}] \rightarrow \mathfrak{R}_+$.

The setting is modelled as a simultaneous-move game involving the third-party developer and firms. The third-party developer chooses a design ϕ and a fee f (for a firm to be able to use ϕ) in order to maximize its profit given firms' expected adoption decisions. Firms make adoption decisions that are optimal given their expectations on (ϕ, f) . After adoption decisions are made, the demand state is realized and firms choose prices. If a firm adopted ϕ then it prices at $\phi(a)$, and if a firm did not adopt ϕ then it chooses an optimal price that does not condition on a . The solution concept is Nash equilibrium.²⁰

Given (ϕ, f) , the third-party developer's profit (or, equivalently, revenue, as there is assumed to be no marginal cost to selling a pricing algorithm) is simply the number of firms that buy ϕ multiplied by f . As explicitly shown in Harrington (2022b) for the duopoly case, the third-party developer's optimal choice is characterized as follows. First, the fraction of firms that will adopt the pricing algorithm, which is denoted θ , is conjectured. I elaborate below how θ may be determined. Second, the third-party developer designs ϕ to maximize the willingness-to-pay (WTP) for an adopter, conditional on a fraction θ of firms adopting. Third, the third-party developer chooses f to equal the WTP (as that will maximize its profit) and results in a fraction θ of firms adopting.

Regarding the determination of θ , if adoption is only constrained by a firm finding it profitable to adopt then the third-party developer will choose θ to maximize its profit taking into account how it will affect the fee it can charge, which is the assumption made in Harrington (2022b). Alternatively, some firms may, for some unmodelled reason, not be willing to delegate their pricing to an algorithm, which then puts an upper bound on θ . Or firms could have some heterogenous firm-specific adoption cost - perhaps from integrating it into their system - which could result in some firms adopting and some not. Or the third-party developer may require marketing and training for adoptions to occur which could limit how fast it sells the pricing algorithm, which would also constrain θ . As the analysis will focus on how the design decision depends on θ , I can be agnostic about the determination of θ . What is critical is that the pricing algorithm is designed by the third-party developer for when a fraction θ of firms adopt it.

The model here differs from that in Harrington (2022b) in two ways. First, as has been mentioned, there is a continuum of firms rather than two firms. Second, the model in Harrington (2022b) allows an adopter to observably commit to its pricing rule prior to non-adopters choosing their prices which meant it acted as a price leader. That commitment effect is not present in the current model. I do not believe commitment is essential to the third-party developer's design decision, nor is it clear which assumption is more appropriate.

²⁰In taking an equilibrium approach, it is implicitly assumed parties' learning algorithms converge to an equilibrium outcome. While that is an admittedly heroic assumption, an equilibrium approach is nevertheless useful in identifying some fundamental incentives determining firms' prices and the third party's design of the pricing algorithm, and that can advance our understanding of the potential effect of a firm outsourcing its pricing decisions to a third party.

I chose not to allow commitment here because it simplifies the analysis. Furthermore, when market variation σ^2 is not too low, the key properties of the pricing algorithm in Harrington (2022b) and here (when adoption decisions are independent) are identical so the change in these two assumptions does not alter the operative forces in the model. In sum, no commitment is an assumption of convenience which is believed to be without consequence for our results, and the assumption of many firms allows us to explore comparative statics with respect to the adoption rate.

In preparation for the analysis in Section 3, I offer some preliminary results here. Given the linearity of demand, an optimal pricing algorithm will be an affine function of a : $\phi(a) = \alpha + \gamma a$ for some (α, γ) .²¹ Given a fraction θ of firms adopt $\alpha + \gamma a$, let us characterize the price of the remaining $1 - \theta$ firms who are non-adopters. Given the pricing algorithm of the adopters, the non-adopters are assumed to choose an equilibrium (uniform) price:

$$\begin{aligned} p^{NA} &\equiv \arg \max_p \int (p - c) (a - bp + d(\theta(\alpha + \gamma a) + (1 - \theta)p^{NA})) G'(a) da \\ &= \arg \max_p (p - c) (\mu - bp + d(\theta(\alpha + \gamma \mu) + (1 - \theta)p^{NA})). \end{aligned}$$

The first-order condition (FOC) is:

$$\mu - bp^{NA}(\theta) + d(\theta(\alpha + \gamma \mu) + (1 - \theta)p^{NA}) - bp^{NA} + bc = 0,$$

which we can solve for the symmetric equilibrium non-adopters' price when a fraction θ of firms adopt the pricing algorithm $\alpha + \gamma a$:

$$p^{NA} = \frac{\mu + bc + d\theta(\alpha + \gamma \mu)}{2b - d(1 - \theta)}.$$

3 Design of the Pricing Algorithm

3.1 Coordinated Adoption Decisions

We begin with the case of coordinated adoptions which is the situation of concern for competition authorities and is the claim of plaintiffs in recent private litigation. Suppose a fraction θ of firms have agreed to adopt the third party's pricing algorithm but only if all do so. The role of the third party can be described in either of two ways. First, it is approached by the colluding firms who instruct the third party to design the pricing algorithm to maximize their joint profit (and the third party will be properly compensated). Second, the third party is knowledgeable of the agreement and designs the pricing algorithm to maximize the WTP of the adopting firms. As their adoption decisions are joint - either all adopt or none adopt - their WTP for the pricing algorithm is an adopter's profit when a fraction θ of firms adopt minus a non-adopter's profit when no firms adopt. With either motivation, the third-party developer designs the pricing algorithm - that is, chooses values for (α, γ)

²¹As shown in Section 3.1, the solution must be affine when firms make coordinated adoption decisions. For the case of independent adoption decisions in Section 3.2, I focus on the case of an affine solution. Though non-affine solutions have not been dismissed as a possibility, their existence is unlikely.

- to maximize adopting firms' joint profit. When adoption decisions are coordinated, the third-party developer's design problem is then:

$$\max_{(\alpha, \gamma)} \int (\alpha + \gamma a - c) (a - b(\alpha + \gamma a) + d(\theta(\alpha + \gamma a) + (1 - \theta)p^{NA})) G'(a) da - \pi^N. \quad (2)$$

π^N is the Nash equilibrium profit when no firm adopts and, as it does not depend on (α, γ) , maximizing the WTP in (2) is the same as maximizing aggregate adopters' profit or, equivalently, an adopter's profit.²² In other words, the third-party developer acts like a cartel manager where the cartel encompasses a fraction θ of firms.

Theorem 1 *Under coordinated adoptions, if a fraction θ of firms adopt then the third-party developer's pricing algorithm is*

$$\alpha = \frac{(b - d\theta)c(2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta)) + d(1 - \theta)(2b(\mu + bc) - d\theta(\mu + (b + d\theta)c))}{2(b - d\theta)(2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta))}$$

$$\gamma = \frac{1}{2(b - d\theta)}.$$

Proof. The third-party developer's optimal pricing algorithm is one that maximizes an adopter's profit given the expected price of the non-adopters. If non-adopters are expected to choose the uniform price p^{NA} then, for any a , the pricing algorithm is designed to set price to maximize:

$$(p - c) (a - bp + d(\theta p + (1 - \theta)p^{NA})).$$

Note that p is chosen by all adopting firms, which reflects their joint adoption decisions. The FOC is:

$$0 = a - 2(b - d\theta)p + d(1 - \theta)p^{NA} + (b - d\theta)c,$$

which can be solved for price to yield:

$$p^C = \frac{a + (b - d\theta)c + d(1 - \theta)p^{NA}}{2(b - d\theta)}. \quad (3)$$

The superscript C refers to "coordinated" adoption decisions.

The non-adopters will settle on an equilibrium price given their expectation of the adopters' price $E_a[p^C]$.²³ p^{NA} is defined by:

$$p^{NA} = \arg \max (p - c) (\mu - bp + d(\theta E_a[p^C] + (1 - \theta)p^{NA}))$$

with FOC:

$$0 = \mu - bp^{NA} + d(\theta E_a[p^C] + (1 - \theta)p^{NA}) - bp^{NA} + bc \Leftrightarrow$$

²²Note that p^{NA} is taken as fixed. Thus, the third-party developer does not act as a leader by supposing its choice of (α, γ) will affect p^{NA} . This is the assumption of no commitment.

²³It is implicitly assumed non-adopting firms know rival firms' average price. This knowledge could occur through learning α , γ , and θ or observing rival firms' prices with a learning dynamic that converges to the equilibrium.

$$p^{NA} = \frac{\mu + bc + d\theta E_a[p^C]}{2b - d(1 - \theta)}. \quad (4)$$

From (3), we have:

$$E_a[p^C] = \frac{\mu + (b - d\theta)c + d(1 - \theta)p^{NA}}{2(b - d\theta)}. \quad (5)$$

Using (4), substitute for p^{NA} in (5):

$$E_a[p^C] = \frac{\mu + (b - d\theta)c + d(1 - \theta) \left(\frac{\mu + bc + d\theta E_a[p^C]}{2b - d(1 - \theta)} \right)}{2(b - d\theta)},$$

and solve for $E_a[p^C]$:

$$E_a[p^C] = \frac{2b(\mu + (b - d\theta)c) + d^2\theta(1 - \theta)c}{2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta)}. \quad (6)$$

Inserting (6) into (4), a non-adopter's price is:

$$p^{NA} = \frac{\mu + bc + d\theta \left(\frac{2b(\mu + (b - d\theta)c) + d^2\theta(1 - \theta)c}{2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta)} \right)}{2b - d(1 - \theta)} = \frac{2b(\mu + bc) - d\theta(\mu + (b + d\theta)c)}{2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta)}. \quad (7)$$

Using (7) in (3), an adopter's price is:

$$\begin{aligned} p^C &= \frac{a + (b - d\theta)c + d(1 - \theta) \left(\frac{2b(\mu + bc) - d\theta(\mu + (b + d\theta)c)}{2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta)} \right)}{2(b - d\theta)} \\ &= \frac{(b - d\theta)c(2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta)) + d(1 - \theta)(2b(\mu + bc) - d\theta(\mu + (b + d\theta)c))}{2(b - d\theta)(2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta))} \\ &\quad + \left(\frac{1}{2(b - d\theta)} \right) a, \end{aligned}$$

which proves the theorem. ■

Let us derive some properties of the pricing algorithm in Theorem 1. From the proof of Theorem 1, we find that, on average, adopters price higher than non-adopters:

$$\begin{aligned} &E_a[p^C] - p^{NA} \\ &= \frac{2b(\mu + (b - d\theta)c) + d^2\theta(1 - \theta)c}{2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta)} - \frac{2b(\mu + bc) - d\theta(\mu + (b + d\theta)c)}{2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta)} \\ &= \frac{d\theta(\mu - (b - d)c)}{2b(2b - (1 + \theta)d) + d^2\theta(1 - \theta)} > 0. \end{aligned}$$

Due to the recommendations of the third-party developer, adopters are pricing to maximize their joint profit, while each non-adopter is pricing to maximize its own profit. Given the adopters are setting supracompetitive prices, a non-adopter maximizes its profit by setting its uniform price to undercut adopters' average price.

Next we turn to assessing the effect of the fraction of firms that adopt the pricing algorithm. The average price for adopters and non-adopters are both increasing in the adoption rate θ :

$$\frac{\partial E_a[p^C]}{\partial \theta} = \frac{2bd(2b - d + 2d\theta)(\mu - (b - d)c)}{(4b^2 - 2bd(1 + \theta) + d^2\theta(1 - \theta))^2} > 0$$

$$\frac{\partial p^{NA}}{\partial \theta} = \frac{d^2\theta(4b - d\theta)(\mu - (b - d)c)}{(4b^2 - 2bd(1 + \theta) + d^2\theta(1 - \theta))^2} > 0.$$

As more firms agree to adopt the pricing algorithm, the third-party developer designs the pricing algorithm to charge higher prices because the cartel of adopting firms is more inclusive; hence, there are fewer non-adopting firms acting as a competitive constraint. In response to higher prices for adopters, the non-adopters price higher, too.

The preceding properties are summarized in Corollary 2.

Corollary 2 *Under coordinated adoptions: i) the average price of adopters and non-adopters are increasing in the adoption rate; and ii) on average, the price of adopters exceeds the price of non-adopters.*

Having characterized the third party's pricing algorithm under coordinated adoptions, let us conclude with a brief discussion of how such coordination may occur. The canonical device is express communication, either between the firms and the third party or with the third party acting as a hub and engaging in bilateral communication with each firm to achieve a common understanding that all are to adopt the pricing algorithm. A method not involving express communication is for the third party to commit to a supply or fee schedule that necessarily implies joint adoption among firms. One scheme is to offer firms a contract that states the pricing algorithm will be supplied only when some minimum participation threshold is satisfied. Though there is no express invitation to collude, such a scheme would seem to meet the evidentiary standard for proving an antitrust violation as it is equivalent to conveying to each firm: "I will sell my pricing algorithm to you if and only if enough other firms also buy it." That is inviting an agreement and a firm's acceptance of such a contract is acceptance of that invitation. As such a contract would seem to be sure to draw antitrust scrutiny, a more subtle scheme is for the third party to have a fee schedule that makes the fee charged to a firm decreasing in how many firms adopt. While subsuming the first scheme (where there is a reasonable fee when some minimum fraction of firms adopt and an exorbitant fee otherwise), the fee schedule could be set so as not to be so blatantly anticompetitive.²⁴

²⁴While our analysis does not consider the option of firms colluding on their own, there are variety of advantages from engaging an IT service provider. First and foremost, collusion is more profitable because firms can achieve the first-best monopoly price discrimination solution. There are other advantages which are common to any hub-and-spoke cartel. Garrod, Harrington, and Olczak (2021) describe how the hub - such as an upstream supplier (e.g., an IT service provider) - can aid in coordinating on an outcome, monitoring for compliance with that outcome, and punishing for non-compliance. When the third party is supplying a pricing algorithm, these factors could manifest as coordinating adoptions through their communications with firms, monitoring firms that they charge the pricing algorithm's recommended price, and punishing by cancelling a firm's subscription to the pricing algorithm (and thereby denying it the ability to price discriminate). One disadvantage from colluding firms involving an IT service provider relates to enforcement.

3.2 Independent Adoption Decisions

Now suppose firms make independent adoptions. This means that a firm anticipates its adoption decision will not affect the adoption decisions of other firms. Focusing on affine pricing algorithms, the third-party developer's design problem is to choose the design parameters (α, γ) to maximize a firm's WTP for the pricing algorithm:²⁵

$$\begin{aligned} \max_{(\alpha, \gamma)} \quad & \int (\alpha + \gamma a - c) (a - b(\alpha + \gamma a) + d(\theta(\alpha + \gamma a) + (1 - \theta)p^{NA})) G'(a) da \quad (8) \\ & - \int (p^{NA} - c) (a - bp^{NA} + d(\theta(\alpha + \gamma a) + (1 - \theta)p^{NA})) G'(a) da. \end{aligned}$$

The first term is the expected profit of adopting the pricing algorithm $\alpha + \gamma a$ and the second term subtracted from it is the expected profit from not adopting and pricing at p^{NA} (and not conditioning price on the demand state). Note that the third party's pricing algorithm affects both terms. For example, if (α, γ) is increased so that prices are closer to the monopoly level, that will raise the profit from adopting (first term) but also the profit from not adopting (second term). As adoption decisions are independent (and there are many firms), the adoption rate for the industry is θ in both cases.²⁶ A property of particular importance is that maximizing the WTP is not equivalent to maximizing an adopter's profit. This means that, in maximizing its profit from selling the pricing algorithm, the third-party developer will not act as a cartel manager. What it implies for the design of the pricing algorithm is described in the next theorem.

Theorem 3 *Under independent adoptions, if a fraction θ of firms adopt then the third-party developer's pricing algorithm is*

$$\begin{aligned} \alpha &= \frac{2bc(b - d\theta) + d\mu(1 - 2\theta)}{2(b - d\theta)(2b - d)} \\ \gamma &= \frac{1}{2(b - d\theta)}. \end{aligned}$$

Proof. For the objective in (8), consider the FOC with respect to α :²⁷

$$\frac{\partial}{\partial \alpha} = \theta (\mu - (b - d\theta)(\alpha + \gamma\mu) + d(1 - \theta)p^{NA}) - \theta (\alpha - c) (b - d\theta) - \theta \gamma \mu (b - d\theta) - \theta (p^{NA} - c) d\theta = 0,$$

Generally, cartel membership is a closely-guarded secret, but an IT service provider's customer list is discoverable if not public information. Thus, it could be easier for a competition authority to determine who is part of an agreement when that agreement involves an IT service provider than when it only involves the firms themselves.

²⁵In (8), p^{NA} refers to the non-adopter's price under independent adoptions and will differ from the expression for p^{NA} in the case of coordinated adoptions.

²⁶With literally an infinite number of firms, an individual firm's adoption decision does not affect the adoption rate. I consider that as an approximation for when there are many finite firms and one firm's decision has a small effect on the adoption rate. Assuming this effect is zero simplifies the analysis while maintaining the key driving forces of the model.

²⁷It is straightforward to show that second-order conditions are satisfied.

and solve for α :

$$\alpha = \frac{\mu + bc + dp^{NA} - 2d\theta p^{NA} - 2b\gamma\mu + 2d\theta\gamma\mu}{2b - 2d\theta}. \quad (9)$$

Consider the FOC with respect to γ :

$$\frac{\partial}{\partial \gamma} = \theta (\sigma^2 + \mu^2 + bc\mu + dp\mu - 2b\alpha\mu - 2b\sigma^2\gamma - 2b\gamma\mu^2 + 2d\theta\sigma^2\gamma + 2d\theta\gamma\mu^2 - 2dp\theta\mu + 2d\theta\alpha\mu) = 0. \quad (10)$$

Use (9) to substitute for α in (10) and then solve for γ :

$$\gamma = \frac{1}{2(b - d\theta)}. \quad (11)$$

Substitute (11) for γ in (9) and solve for α :

$$\alpha = \frac{bc + dp^{NA} - 2d\theta p^{NA}}{2b - 2d\theta}.$$

We then have three unknowns (α, γ, p^{NA}) and three equations:

$$\begin{aligned} \alpha &= \frac{bc + dp^{NA} - 2d\theta p^{NA}}{2b - 2d\theta} \\ \gamma &= \frac{1}{2(b - d\theta)} \\ p^{NA} &= \frac{\mu + bc + d\theta(\alpha + \gamma\mu)}{2b - d(1 - \theta)}. \end{aligned}$$

The solution to this system comprises the values for (α, γ) in the statement of the theorem and

$$p^{NA} = \frac{\mu + bc}{2b - d}.$$

The third-party developer's pricing algorithm is then

$$p^I(a) = \frac{2bc(b - d\theta) + d\mu(1 - 2\theta)}{2(b - d\theta)(2b - d)} + \left(\frac{1}{2(b - d\theta)} \right) a,$$

where the superscript I refers to "independent" adoption decisions. ■

A crucial property of the pricing algorithm is that the average price it produces is the same as the Nash equilibrium price without a third-party developer and, consequently, is independent of the adoption rate:

$$E_a [p^I] = \frac{2bc(b - d\theta) + d\mu(1 - 2\theta)}{2(b - d\theta)(2b - d)} + \left(\frac{1}{2(b - d\theta)} \right) \mu = \frac{\mu + bc}{2b - d}.$$

Similarly, the non-adopters' price is also $\frac{\mu + bc}{2b - d}$. Hence, when firms make independent adoption decisions, there is no anticompetitive effect with regards to the average price; competitors

using the third-party developer's pricing algorithm price the same, on average, as firms in the absence of a third-party developer.²⁸

Corollary 4 *Under independent adoptions: i) the average price of adopters and non-adopters are independent of the adoption rate; and ii) on average, the price of adopters equals the price of non-adopters.*

3.3 Comments

Note that an adopter's average price is higher with coordinated adoptions than with independent adoptions:

$$E_a[p^C] = \frac{2b(\mu + (b - d\theta)c) + d^2\theta(1 - \theta)c}{2b(2b - d(1 + \theta)) + d^2\theta(1 - \theta)} > \frac{\mu + bc}{2b - d} = E_a[p^I].$$

However, the sensitivity of price to the demand state is the same:

$$\frac{\partial p^I}{\partial a} = \frac{1}{2(b - d\theta)} = \frac{\partial p^C}{\partial a}.$$

Furthermore, this sensitivity is greater when the adoption rate is higher:

$$\frac{\partial \left(\frac{1}{2(b - d\theta)} \right)}{\partial \theta} = \frac{d}{2(b - d\theta)^2} > 0.$$

These properties are explained in the next section.

In concluding this section, let us address the matter of the optimal adoption rate. Substituting the pricing algorithm from Theorem 3 into the third-party developer's objective function in (8), one can simplify it to find the WTP of a firm for the pricing algorithm is:

$$\frac{\theta\sigma^2}{4(b - d\theta)}.$$

Note that it is increasing in the adoption rate:

$$\frac{\partial \left(\frac{\theta\sigma^2}{4(b - d\theta)} \right)}{\partial \theta} = \frac{b\sigma^2}{4(b - d\theta)^2} > 0.$$

Thus, the third-party developer designs the pricing algorithm so that adoptions are strategic complements: the more firms who are expected to adopt the pricing algorithm, the more attractive it becomes to adopt it.

²⁸Even if firms' beliefs on the adoption rate differ from the actual adoption rate, Theorem 3 implies the average price is independent of the actual adoption rate. Theorem 3 characterizes the pricing algorithm for the third party that maximizes firms' WTP which depends on what firms believe is the adoption rate. Thus, θ in Theorem 3 is the adoption rate believed by firms when considering whether to adopt. Since the associated average price for adopters is $\frac{\mu+bc}{2b-d}$ and the uniform price for non-adopters is also $\frac{\mu+bc}{2b-d}$ then, regardless of the actual adoption rate, the average price of all firms is $\frac{\mu+bc}{2b-d}$.

If an adopter incurs no other cost than the fee charged by the third-party developer, the third-party developer will optimally charge a fee equal to the WTP of $\frac{\theta\sigma^2}{4(b-d\theta)}$. Thus, its profit from selling the pricing algorithm to a fraction θ of firms is

$$\theta \left(\frac{\theta\sigma^2}{4(b-d\theta)} \right)$$

which is increasing in the adoption rate:

$$\frac{\partial \left(\frac{\theta^2\sigma^2}{4(b-d\theta)} \right)}{\partial \theta} = \frac{(2b-d\theta)\theta\sigma^2}{4(b-d\theta)^2} > 0.$$

Hence, the third-party developer optimally designs and prices the pricing algorithm so that all firms adopt it.

Complete adoption might not occur for the various reasons mentioned earlier: the presence of some firm-specific adoption cost which is heterogeneous across firms, limitations on how fast the third-party developer can install the pricing algorithm, and some firms' resistance to delegating pricing authority (perhaps out of fear of antitrust litigation, as is occurring). Even if not all firms adopt, it will always be optimal for the third-party developer to design the pricing algorithm to maximize the WTP as that will maximize the price it can charge to the firms who do adopt.

4 Comparison of Third-Party Pricing Algorithms under Coordinated and Independent Adoptions

As shown in Section 3, if firms' decisions to adopt a third-party developer's pricing algorithm are independent then adopters' and non-adopters' average prices will not depend on how many firms adopt, and the average price of adopters and non-adopters will be the same. If instead firms' adoption decisions are coordinated then both adopters' and non-adopters' average prices will be higher when more firms adopt, and the average price of adopters will exceed the average price of non-adopters. In this section, the source of these differences is explained and their generality is discussed.²⁹

Let us consider three scenarios under which a fraction θ of firms condition price on the demand state. $p^C(a)$ is the third party's pricing algorithm when firms are making coordinated adoption decisions. It is designed to maximize the aggregate profit of adopters or, equivalently, an adopter's profit. $p^I(a)$ is the third party's pricing algorithm when firms are making independent adoption decisions. It is designed to maximize the willingness-to-pay of an adopter which, with independent adoption decisions, is the profit from adopting minus the profit from not adopting while holding fixed that a fraction θ of rival firms are adopting. $p^F(a)$ is the pricing algorithm when a firm designs the algorithm itself and firms do so independently. Its closed-form solution is in (1). It is designed to maximize the firm's profit, given a fraction θ of rival firms are using $p^F(a)$, and is the pricing algorithm that

²⁹The underlying forces are the same as in Harrington (2022b) for the duopoly case though a different approach is used to explain them.

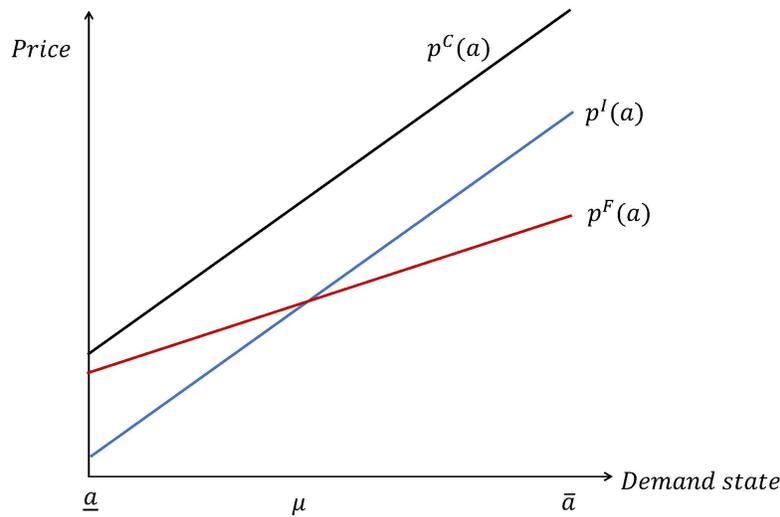
would occur under competition if the firms could design it on their own. As summarized in Table 1, these pricing algorithms differ in terms of the identity of the designer and the relationship between firms' adoption decisions

Table 1

	Firms' adoption decisions	
Designer of pricing algorithm	Independent	Coordinated
Third party	p^I	p^C
Firm	p^F	-

The three pricing algorithms are depicted in Figure 1.

Figure 1



Let us compare these pricing algorithms. To begin, p^I has the same average price as p^F which is less than the average price of p^C ,

$$E_a [p^C] > E_a [p^I] = \frac{\mu + bc}{2b - d} = E_a [p^F],$$

but p^I is more sensitive to the demand state than p^F ,

$$\frac{\partial p^I}{\partial a} = \frac{1}{2(b - d\theta)} > \frac{1}{2b - d\theta} = \frac{\partial p^F}{\partial a}.$$

Next note that the third party's pricing algorithm has price respond to the demand state in the same way whether adoption decisions are independent or coordinated:

$$\frac{\partial p^I}{\partial a} = \frac{1}{2(b - d\theta)} = \frac{\partial p^C}{\partial a}.$$

As depicted in Figure 1, p^I is a parallel shift down of p^C .

Let me explain why the third party designs p^I to have these properties when firms' adoption decisions are independent. The third party chooses the design that maximizes

1
2
3 firms' WTP which is the profit from adopting minus the profit from not adopting, given
4 a fraction θ of firms are anticipated to adopt. Towards finding the best design, consider
5 the thought experiment of creating a supracompetitive pricing algorithm by having price be
6 higher than the competitive pricing algorithm p^F at each demand state, so it is $p^F(a) + \varepsilon(a)$
7 where $\varepsilon(a)$ is positive (and small enough that price does not exceed the monopoly price).
8 Compared to $p^F(a)$, $p^F(a) + \varepsilon(a)$ raises profit both from adopting and not adopting because
9 rival firms - specifically, those who adopt - will be setting higher prices. By adopting itself,
10 the firm sets those same high prices but, and this is the key point, by not adopting the firm
11 would set an average (more specifically, uniform) price that is the best response to adopters'
12 high average price. That best response is below the average price of the pricing algorithm
13 because the pricing algorithm is setting price above the competitive pricing algorithm p^F .
14 Summing up, a pricing algorithm with a higher average price than p^F will raise both the
15 profit from adopting and not adopting - because the adopting rivals of a firm now have a
16 higher average price - but it raises the profit from not adopting *more* because a non-adopter
17 sets the optimal average price in response to adopters' higher average price. Consequently,
18 $p^F(a) + \varepsilon(a)$ lowers the WTP compared to $p^F(a)$. The general takeaway is that a third party
19 will not want to raise the average price above the competitive pricing algorithm, which is
20 why we find $E_a [p^I] = E_a [p^F]$.

21
22 Under the assumption of independent adoption decisions, a third party's optimal pricing
23 algorithm will then set average price at the competitive level. In contrast, if adoption
24 decisions are coordinated then the third party will maximize the profit from adopting -
25 rather than the difference between adopting and not adopting - and that necessarily means
26 a higher average price. With coordinated adoptions, the situation is equivalent to a partial
27 cartel which encompasses a fraction θ of firms where the third party is acting like a cartel
28 manager when it selects p^C . The higher is θ , the more inclusive is the cartel which implies
29 p^C is higher and, therefore, average price is higher (as there are fewer non-cartel members
30 to undercut the high average price of adopters). Thus, when adoptions are coordinated, p^C
31 has average price increasing in the adoption rate θ , while with independent adoptions, the
32 pricing algorithm's average price is at the competitive level and thus does not depend on θ .

33
34 For when firms' adoption decisions are independent, I have just explained the third party
35 will not have the pricing algorithm set a supracompetitive average price like a cartel manager
36 would. However, it *will* have price respond to the demand state as a cartel manager would.
37 p^I is designed to have price respond to the demand state in order to maximize an adopter's
38 profit, which means making price more sensitive to the demand state than is the case under
39 competition.³⁰ Increasing the sensitivity of price to the demand state raises the profit from
40 adopting and, most importantly, does *not* affect the profit from not adopting (as long as
41 the average price is unchanged). The reason is that a non-adopter's profit depends only on
42 the average price of adopters, not how adopters respond to the demand state. Though in
43 response to a high demand state, a non-adopter would like to set a high price that undercuts
44 the high price of adopters, it is incapable of doing so because only by having the third party's

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³⁰The reason that a cartel manager would make price more sensitive to the demand state is as follows.
When the demand state is stronger, a firm's profit-maximizing price is higher. Given firms' prices are
strategic complements, a firm optimally raises price more when other firms also raise their prices. A cartel
manager will take that effect into account so firms' prices will rise more in response to stronger demand
compared to when prices are set independently.

pricing algorithm can it condition price on the demand state. In the case of independent adoptions, the secret to the third party maximizing the WTP (= profit from adopting minus the profit from not adopting) is to have price respond to the demand state in a way that maximizes the profit from adopting, while not raising the average price so the profit from not adopting does not rise.

Corollaries 2 and 4 follow from these properties of the pricing algorithms. Under independent adoption decisions, the third party maximizes the WTP by keeping the average price fixed compared to when there is no third party - in order to avoid raising the profit from not adopting - which implies an adopter's average price is independent of the adoption rate. Under coordinated adoption decisions, a third party maximizes adopters' profit and that implies a higher average price. The more firms that adopt, the stronger is that effect as the "adopters' cartel" is more inclusive so there are fewer non-adopting firms to undercut the adopters' supracompetitive average price. Hence, average price is increasing in the adoption rate when firms' adoption decisions are coordinated.

Though the model is highly structured with linear demand and symmetric firms, the forces determining the properties of the pricing algorithms seem quite general. This is clearly the case with coordinated adoptions as the third party is acting as a cartel manager - designing the pricing algorithm to maximize adopters' profits - and that will necessarily result in a higher average price when the cartel is more inclusive (i.e., the adoption rate is higher). Less immediate is the robustness of the third party's pricing algorithm under independent adoptions: average price is at the competitive level and is independent of the adoption rate. It is certainly possible that average price may not be exactly at the competitive level with non-linear demand functions and asymmetric firms. However, there is no reason to expect average price to systematically depend on the adoption rate. As explained above, the properties of the pricing algorithm under independent adoptions are based on the ability of a non-adopter to optimize with respect to the average price of adopters but not to how adopters' respond to the demand state. Those features are general and not specific to the particular assumptions of the model.

A second issue related to robustness is that we have only examined when either all adoption decisions are independent or all adoption decisions are coordinated. Suppose a fraction θ of firms adopt the pricing algorithm but only a fraction ω of them coordinate their adoption decisions. It seems reasonable to expect the third-party developer's optimal pricing algorithm to be some mixture of the optimal pricing algorithm when all firms make coordinated adoption decisions, p^C , and the optimal pricing algorithm when all firms make independent decisions, p^I . If that is true then the properties in Corollary 2 would still hold, it would just be the effect is not as strong. I would then expect the average price of adopters to be increasing in the adoption rate as long as some (or all) of the adoptions decisions are coordinated. While this evidence would tell us there is an agreement between the third-party developer and some adopters, it would not tell us whether the agreement encompasses all adopters or, when it does not encompass all adopters, which adopters are part of the agreement. Nevertheless, with evidence of an agreement, other evidence could be used to determine which adopters are party to the agreement.

5 An Economic Test for an Unlawful Agreement

The preceding analysis provides us with three candidate tests for providing evidence of an agreement between a third-party developer who designed a pricing algorithm and the firms who adopted it.

- Test #1: The average price of adopters is increasing in the adoption rate.
- Test #2: The average price of non-adopters is increasing in the adoption rate.
- Test #3: The average price of adopters exceeds the average price of non-adopters.

Test #1 is at the core of the incentives of a third-party developer when it comes to the design of the pricing algorithm. When adopters have an agreement to adopt the pricing algorithm, the third-party developer designs the pricing algorithm to maximize adopters' profit which implies supracompetitive prices. This supracompetitive markup is greater when more firms are part of the agreement which implies the average price of adopters is increasing in the adoption rate. This relationship is not predicted when firms make independent adoption decisions in which case adopters' average price is independent of the adoption rate.³¹

Test #2 is an implication of the conduct underlying Test #1. Given firms with an agreement to adopt the pricing algorithm set supracompetitive prices, non-adopters will optimally respond with higher prices (though, on average, undercutting adopters' prices). As the adopters' supracompetitive markup is higher with a higher adoption rate, non-adopters' supracompetitive markup is then also higher with a high adoption rate. A potential concern with this test is if non-adopters take time to learn the optimal response to adopters' prices, so adopters' average price is higher but non-adopters' average price has not yet responded to that higher price. Thus, it is possible there is an agreement which is picked up by Test #1 but not by Test #2. For this reason, Test #1 is superior to Test #2.

A possible concern with Test #3 is that it may not be robust to firm asymmetries. For example, suppose firms have different marginal costs and the pricing algorithm allows a firm to program in its marginal cost. While all adopters set supracompetitive prices, an adopter with a lower cost sets a lower price than an adopter with a higher cost. Further suppose, for whatever reason, a firm is more likely to adopt the pricing algorithm if its cost is lower. Even if firms have an agreement and the pricing algorithm is designed as the theory predicts, adopters' average price may not be higher than non-adopters' average price because

³¹It has been assumed that non-adopting firms are competing which seems reasonable since it isn't clear why a third party would be involved in an agreement that did not require participating firms to use its pricing algorithm. Regardless, consider some or all non-adopting firms being part of the agreement and let us distinguish between the adoption rate (i.e., the fraction of firms using the pricing algorithm) and the agreement rate (i.e., the fraction of firms who are colluding). The analysis of this paper assumes those two rates are the same but Test #1 should be effective as long as they move together. The only situation in which the test fails is if the agreement rate is independent of the adoption rate. For example, if all firms are part of the agreement then the third party will optimally design the pricing algorithm to be the monopoly pricing function and the non-adopting firms will set the monopoly uniform price. Consequently, the average price will equal the average monopoly price which does not depend on the adoption rate. That special case aside, the proposed test should work even when non-adopting firms are part of the agreement as long as the more firms that adopt the pricing algorithm, the more firms are part of the agreement.

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3 the marginal cost of adopters is lower. In that case, Test #3 would not be satisfied even
4 though there is an agreement. In conclusion, Test #1 is the most general and robust of the
5 three tests and is the one recommended.³²
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7 When there is not direct evidence of communication supporting an agreement, evidence
8 is circumstantial and obtaining a conviction often requires both economic and non-economic
9 evidence. The test proposed here offers an approach to delivering economic evidence. Based
10 on the European Court of Justice's recent *Eturas* decision, let me describe how this evidence
11 along with certain non-economic evidence could be used to prove a violation of TFEU 101(1).
12 In *Eturas*, travel agencies (firms) offered their services on the website of Eturas (third party)
13 who sent them all a message announcing it had imposed a cap on the discounts that they
14 could offer (though it was possible for an agency to circumvent the cap). The ECJ concluded
15 that "those economic operators may - if they were aware of that message - be presumed to
16 have participated in a concerted practice ... unless they publicly distanced themselves from
17 that practice, reported it to the administrative authorities or adduce other evidence to rebut
18 that presumption, such as evidence of the systematic application of a discount exceeding the
19 cap in question."³³ In that case, the third party imposed the practice and the ECJ viewed
20 an informed firm as engaging in that practice unless they took clear action against it. In
21 the case of a third party offering a supracompetitive pricing algorithm, an adopting firm is
22 more culpable because the pricing algorithm is not imposed on them; a firm had to take
23 explicit action to adopt it and, in fact, paid for it. Thus, if firms knew the third party's
24 pricing algorithm was made available to all firms and it was designed to reduce competition
25 (as reflected in a higher average price when more firms adopt) then, with their adoption of
26 the pricing algorithm, they can "be presumed to have participated in a concerted practice".
27

28 An empirical implementation of Test #1 would involve regressing adopters' prices on the
29 adoption rate. The data required are: 1) whether (and possibly when) a firm adopted the
30 third-party developer's pricing algorithm; 2) prices of the firms who adopted; and 3) other
31 variables that could affect observed prices. At a minimum, a competition authority will need
32 to know which firms adopted and their prices. If not publicly available, it is straightforward
33 to obtain this information from the third party and adopting firms through the legal discovery
34 process. The data set will need to have variation in the adoption rate, which could be across
35 time or markets. Inter-market variation could come from a third-party developer offering
36 its pricing algorithm in different geographic markets. One can use the variation in adoption
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43 ³²While these tests are derived from when there is a continuum of firms, I expect them to hold with a finite
44 number of firms. The test is based on two properties: 1) under coordinated adoptions, the average price
45 is increasing in the fraction of adopting firms; and 2) under independent adoptions, the average price is
46 constant with respect to the fraction of adopting firms. (2) is an implication of the property that the average
47 price equals the static Nash equilibrium price without a price discriminating third party. That property
48 is not only true for an infinite number of firms but also for the case of two firms, as shown in Harrington
49 (2022b), and I conjecture it holds for any finite number of firms. Hence, it is generally expected that the
50 average price is independent of the adoption rate under independent adoptions. (1) follows from the optimal
51 cartel price being increasing in the fraction of firms that are cartel members which certainly holds when the
52 number of firms is finite. Thought the tests are not applicable to the duopoly case, that is only because
53 it is not possible to have variation in the adoption rate under coordinated adoptions; all firms must adopt.
54 Contrasting that with when there is a triopoly, the adoption rate can be increased from 2/3 to 1 and it is
55 predicted the average price rises (remains the same) when adoptions are coordinated (independent).

56 ³³Judgment of the Court (Fifth Chamber) of 21 January 2016 - "Eturas" UAB and Others v Lietuvos
57 Respublikos konkurencijos taryba, para. 51.
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3 rates across markets to estimate the relationship between adopters' average price and the
4 adoption rate. Alternatively, there could be a single market with time-series variation in the
5 adoption rate. While the theory is based on simultaneous adoption decisions, it applies as
6 well to sequential adoption decisions as long as the third-party developer adjusts the pricing
7 algorithm to changes in the adoption rate.
8

9 A concern with this empirical approach is the possible endogeneity of the adoption rate.
10 If firms are heterogeneous and that heterogeneity affects both their adoption decisions and
11 the prices set by the pricing algorithm then the estimated coefficient on the adoption rate will
12 be biased. For example, suppose firms differ in marginal cost and firms with higher marginal
13 cost tend to adopt sooner. Even if there is an agreement, it is possible the average price
14 of adopters is higher when the adoption rate is lower because it is the high marginal cost
15 firms that adopt early which, by construction, is when the adoption rate is low. While such
16 bias could occur, variation in firms' adoption decisions need not imply bias. For example,
17 suppose variation in adoption decisions is due to some firm-specific cost to installing a pricing
18 algorithm which varies across firms. Firms with a lower adoption cost are more likely to adopt
19 (or adopt sooner) and that could be the source of variation in adoption rates. As the firm-
20 specific adoption cost does not affect the prices produced by the pricing algorithm, estimates
21 would not be biased. Or suppose the variation in adoption rates is due to the constrained
22 capacity of the third-party developer to sell the pricing algorithm, perhaps due to limited
23 personnel to market and install the program. That could introduce exogenous variation in
24 adoption rates and again allow for unbiased estimates.
25

26 As proof of concept, the empirical test proposed here was conducted in Assad et al (2023)
27 for retail gasoline markets in Germany. In doing so, they addressed the issue of the possible
28 endogeneity of the adoption rate and provided a method for inferring when a firm adopted
29 the pricing algorithm even when that information is confidential. The study found evidence
30 supporting the hypothesis in Test #1: the average price of adopters is higher when more
31 firms adopt.
32

33 In concluding this section, let me address the Lucas critique and explain why the test is
34 likely to be effective even it is known to be used as economic evidence of an agreement. To
35 avoid a positive test result, a third party would have to design the pricing algorithm so the
36 average price is not increasing in the adoption rate. The third party is then not designing
37 the pricing algorithm to maximize firms' WTP which ultimately means having to charge a
38 lower fee and that will lower the third party's profit. Hence, there is a cost to the third
39 party reducing the test's efficacy. In optimally modifying their conduct in response to the
40 test being used by competition authorities and courts, one would expect the third party to
41 trade-off the lower chance of detection and conviction against the lower profit realized with a
42 less optimal pricing algorithm. As it seems unlikely the optimal response is a corner solution
43 which makes the test entirely ineffective, I would expect the test to still have power. The
44 general principle at work is that, even under the Lucas critique, a test has power as long as
45 it is costly for firms to reduce its effectiveness.³⁴
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52 ³⁴This point is made as well in the context of cartel screening; see Section 6.3 of Harrington (2008) where
53 it is also discussed how, in practice, many colluding firms fail to engage in conduct that would *costlessly*
54 make detection less likely. Just because the Lucas critique could undermine a test, it is still an empirical
55 question whether the parties are smart enough to do so.
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6 Concluding Remarks

This study has shown how the properties of a third-party developer's pricing algorithm depend on the manner in which adoption decisions are made by competitors in a market. If firms are independently deciding whether to adopt the pricing algorithm, the average price of adopting firms is predicted to be independent of the number of firms that adopt. If instead firms are coordinating their adoption decisions then the average price of adopting firms is predicted to be increasing in the number of firms that adopt. These findings provide a test for determining whether there is an unlawful agreement between a third-party developer and adopting firms.

The economic evidence provided by this test could be used in conjunction with other evidence to prove a violation of Article 101 of the Treaty on the Functioning of the European Union, Section 1 of the Sherman Act, or some other jurisdiction's competition law pertaining to collusion. In the United States, a case can be dismissed on the grounds that the evidence in a complaint does not make it sufficiently plausible that firms have an unlawful agreement.³⁵ Plaintiffs are then required to provide sufficient evidence prior to being permitted to go to discovery where they could obtain corporate documents and other critical information. The economic evidence delivered by the test developed here could help surmount that plausibility hurdle. Finally, this test could be used to screen possible cases for investigation and thereby avoid wrongly prosecuting software developers who are not engaging in unlawful activity. Evidence that prices are higher when more firms adopt a third party's pricing algorithm is supportive of coordinated conduct and thus an investigation may be warranted.

³⁵ “[O]nly a complaint that states a plausible claim for relief survives a motion to dismiss.” *Ashcroft v. Iqbal*, 556 U.S. 662, 679 (2009). That decision built on *Bell Atlantic Corp. v. Twombly*, 550 U.S. 544 (2007).

7 Appendix

Suppose a fraction $\theta \in [0, 1]$ of firms can condition price on the demand state a and a fraction $1 - \theta$ of firms cannot and thus set a uniform price which is denoted p' . $p^F(a)$ denotes the equilibrium pricing function for those firms who can condition price on the demand state and is defined by:

$$p^F(a) = \arg \max_p (p - c)(a - bp + d(\theta p^F(a) + (1 - \theta)p')).$$

The first-order condition is

$$a - 2bp^F(a) + d(\theta p^F(a) + (1 - \theta)p') + bc = 0,$$

which can be solved for:

$$p^F(a) = \frac{a + d(1 - \theta)p' + bc}{2b - d\theta}. \quad (12)$$

The equilibrium price for a firm that sets a uniform price is defined by:

$$\begin{aligned} p' &= \arg \max_p \int (p - c)(a - bp + d(\theta p^F(a) + (1 - \theta)p'))G'(a)da \\ &= \arg \max_p (p - c)(\mu - bp + d(\theta E_a[p^F(a)] + (1 - \theta)p')). \end{aligned}$$

The first-order condition is

$$\mu - 2bp' + d(\theta E_a[p^F(a)] + (1 - \theta)p') + bc = 0,$$

which can be solved for:

$$p' = \frac{\mu + d\theta E_a[p^F(a)] + bc}{2b - d(1 - \theta)}. \quad (13)$$

Substituting (12) into (13) gives us the expression for p' :

$$p' = \frac{\mu + d\theta \left(\frac{\mu + d(1 - \theta)p' + bc}{2b - d\theta} \right) + bc}{2b - d(1 - \theta)} \Leftrightarrow p' = \frac{\mu + bc}{2b - d}. \quad (14)$$

Using (14) in (12), we have the expression for $p^F(a)$:

$$p^F(a) = \frac{a + d(1 - \theta) \left(\frac{\mu + bc}{2b - d} \right) + bc}{2b - d\theta} = \frac{d(1 - \theta)\mu + (2b - d\theta)bc}{(2b - d\theta)(2b - d)} + \left(\frac{1}{2b - d\theta} \right) a.$$

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